

1.
 - (a) Which do you think is bigger, e^π or π^e ? Make a guess. Do you have a reason for your guess?
 - (b) Let $f(x) = \frac{\ln(x)}{x}$. Find the maximum of $f(x)$.
 - (c) Use this maximum to find and prove which is bigger, e^π or π^e . (Hint: simplify and think about log rules). Was your guess correct?
2. An arbelos is a region enclosed by three mutually tangent semi-circles (two smaller semicircles inside one larger circle, all of which have their centers lying on the same line).
 - (a) Sketch an arbelos where the large circle has radius 1, and label the two smaller radii by t and p . Label the distance between the center of the largest circle and the point where the two smaller circles meet by x . (Try sketching it yourself first, but in case this description is too confusing, there is a sketch on the back of this worksheet that you can check.)
 - (b) Write a function for the area of the arbelos in terms of x , t , and p .
 - (c) Write equations relating x and t and x and p .
 - (d) Substitute so that you have an area function $A(x)$ in terms of x only.
 - (e) Find what x should be to maximize the area of the arbelos.
 - (f) What does this look like on your picture?
3. Let $g(x) = |x|$.
 - (a) For what values of x is $g(x)$ differentiable?
 - (b) Show that $g'(x) = x/|x|$.
 - (c) Use the chain rule to find a formula for $\frac{d}{dx}(|f(x)|)$, and then use it to find the derivative of $h(x) = |x^2 - 4|$ at $x = 1$.
 - (d) Can $\frac{d}{dx}(|f(x)|)$ exist at a point where $f(x) = 0$?
4. A spherical snowball melts at a rate proportional to its surface area. Show that its radius shrinks at a constant rate. (Hints: What does "proportional" mean? What are the formulas for volume and surface area of a sphere in terms of its radius?)