1. Given that $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3} h(x) = 7$, find the following limits, stating the limit rules that you use, or explain why the limit laws do not apply.

$$\text{(a)} \lim_{x\to 3} f(x)h(x) \quad \text{(b)} \lim_{x\to 3}\left(4f(x) + \frac{2}{h(x)}\right)$$
$$\text{(c)} \lim_{x\to 3} \frac{h(x)}{x-1} \quad \text{(d)} \lim_{x\to 3} \frac{f(x)+3}{h(x)-7}$$

2. Evaluate the following limits. State what limit laws or theorems you use.

$$\text{(a)} \lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3} \quad \text{(b)} \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \quad \text{(c)} \lim_{y\to 0} y\sin(y)$$

3. Determine whether the following statements are true or false and give an explanation or counterexample. Assume a and L are finite numbers.
   (a) If $\lim_{x\to a} f(x) = L$, then $f(a) = L$.
   (b) If $\lim_{x\to a^+} f(x) = L$, then $\lim_{x\to a^-} f(x) = L$.
   (c) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = L$, then $f(a) = g(a)$.
   (d) The limit $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist if $g(a) = 0$.
   (e) If $\lim_{x\to 1^+} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^+} f(x)}$, it follows that $\lim_{x\to 1^-} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^-} f(x)}$.

4. Give an example of functions $f(x)$ and $g(x)$ such that $\lim_{x\to 0} (f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

5. (a) Suppose $f(x) = 1$ when x is an irrational number and $f(x) = 0$ when x is rational. Does $f(0)$ and $\lim_{x\to 0} f(x)$ exist? If so what is it?
   (b) Suppose $f(x) = 0$ when x is an irrational number and $f(x) = x^2$ when x is rational. Does $f(0)$ and $\lim_{x\to 0} f(x)$ exist? If so, what is it?
   (c) Further reading: See the attached wikipedia article about Thomae's function. This might also be an interesting example to think about later when we discuss the rigorous definition of a limit and even later when we discuss differentiability.