1. Rolle's Theorem says: Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$. Then there is at least one point c in (a, b) such that $f'(c) = 0$.
   (a) Draw a picture that illustrates Rolle's Theorem. That is, draw a picture where the hypotheses are satisfied and show on your sketch where the conclusion is satisfied. Must there be only one point c that satisfies the conclusion?
   (b) Draw a picture that shows that the condition of continuity is necessary for the theorem to work.
   (c) Draw a picture that shows that the condition of differentiability is necessary for the theorem to work.
2. Determine whether Rolle's Theorem applies to the following functions on the given interval. If so, find the point(s) that satisfy the conclusion of Rolle's Theorem. If Rolle's Theorem does not apply, does that guarantee that there is no point in the interval where the function has slope 0?
   (a) $f(x) = (x - 2)(x - 3)^2$ on $[2, 3]$
   (b) $g(x) = x^2$ on $[-2, 4]$
3. Recall the Extreme Value Theorem (Thorem 4.1 in your book): A function that is continuous on a closed interval $[a, b]$ has an absolute maximum value and an absolute minimum value on that interval.
   Using the Extreme Value Theorem, prove Rolle's Theorem:
   (a) Explain why the Extreme Value Theorem applies here, and what it tells you.
   (b) This proof is most easily done by considering two separate cases. First, do the case where f has both its absolute maximum and minimum values at the endpoints. (Hint: from the hypotheses of Rolle's theorem, you also know that $f(a) = f(b)$. That along with assuming $f(a)$ and $f(b)$ are the minimum and maximum values of f tells you what?)
   (c) Now assume the other case: there is a maximum or minimum value of f that does not occur at an endpoint. What does that tell you?
   (d) End your proof with a nicely written conclusion.
4. The Mean Value Theorem is an extension of Rolle's Theorem. It says: If f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there is at least one point c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
   (a) Draw a picture to illustrate the MVT (draw one where Rolle's theorem is not satisfied).
   (b) Use your picture to describe how you could prove the MVT using Rolle's Theorem. Don't write a formal proof, but explain the basic geometrical idea.
   (c) Use the Mean Value Theorem to design a speed trap: You want to catch someone speeding. The problem is that radar guns aren't working because people have radar detectors (or they just see the police car ahead of them on the side of the road), so they slow down when they approach a police car. You have two police cars that can be positioned at fixed points along a road. Assume that the road is long and straight, the speed limit is 60 mph, and that a car will slow down to 60mph when passing a police car, but will go faster when not near a police car. How can you use the Mean Value Theorem to catch them speeding anyway? (Police really do this). Does this method require radar guns at all? Can you prove they were speeding under all circumstances?