

1. Definition: If  $g'(x) = h(x)$ , then we say  $h(x)$  is an antiderivative of  $g(x)$ .
  - (a) In general, do functions have unique antiderivatives? Can you think of more than one antiderivative for  $f(x) = x^2$ ?
  - (b) Suppose  $g'(x) = h(x)$ . How can you write the most general antiderivative for  $g(x)$ ? (This is usually what people mean when they say the antiderivative of a function.)
  - (c) From the basic definition and all the derivatives you already know, you now know a whole list of antiderivatives. To test the concept, for the following functions, write down the most general antiderivative:
    - i.  $f(x) = \cos(x)$
    - ii.  $f(x) = \frac{1}{1+x^2}$
    - iii.  $f(x) = x^n$  (careful - does this depend on what  $n$  is?)
2. Another way to write "the most general antiderivative of  $f(x)$ " is to write  $\int f(x)dx$ . (We will discuss this notation more later). Find the following indefinite integrals (a.k.a. antiderivatives). You may have to put in a little bit of thought - the idea is to find what function you could differentiate in order to get each of these functions. Remember to check your answer by differentiating.
  - (a)  $\int 2 dx$
  - (b)  $\int (9-x)^2 dx$
  - (c)  $\int \sin(9x+5) dx$
  - (d)  $\int (4\theta + \cos 8\theta) d\theta$
  - (e)  $\int te^{t^2} dt$
3. Suppose  $F$  is an antiderivative of  $f$ , and  $G$  is an antiderivative of  $g$ , that is,  $F'(x) = f(x)$  and  $G'(x) = g(x)$ . For each of the following, show that it must be true or give a counterexample to show it is false.
  - (a) If  $f = g$  then  $F = G$ .
  - (b) If  $F$  and  $G$  differ by a constant, then  $f = g$ .
  - (c) If  $f$  and  $g$  differ by a constant, then  $F = G$ .
4. Suppose that  $F$  is an antiderivative of  $f$ , that is,  $F'(x) = f(x)$ .
  - (a) Show that  $\frac{1}{2}F(2x)$  is the antiderivative of  $f(2x)$ .
  - (b) Find the general antiderivative of  $f(kx)$  for any constant  $k$ .
  - (c) What is the general antiderivative of  $f(kx+a)$  for constants  $k$  and  $a$ ?
5. A car traveling at 84 ft/sec begins to decelerate at a constant rate of 14 ft/sec<sup>2</sup>. After how many seconds does the car come to a stop and how far will the car have traveled before stopping?