1. In this problem you will work out the definition of the integral of a continuous as the area under the curve. You may already know the answer from your book, but the point is to understand the answer. Do the following steps both for a specific function $g(x) = x^2$ on the interval [0,4] and for a general continuous, differentiable function $f(x)$ on the interval [a,b]. (For now assume $f(x)$ is positive, so its graph is above the x-axis.)

   (a) Sketch the function and interval.

   (b) Approximate the area under the function on the given interval by sketching four rectangles with their left-hand basepoint on the graph of the function. Write this approximation as a sum.

   (c) Do the same type of approximation but now with n rectangles. This is the left-hand Riemann Sum.

   (d) Write the exact area under the curve as a limit of the sum.
   Note: When evaluating integrals of general functions, it is important to think about using more general partitions than we have done here. But, as long as $f(x)$ is continuous and differentiable on [a,b], this limit of the left-hand Riemann Sum is equal to the integral $\int_a^b f(x)dx$, and it doesn't matter if we use left, right, or mid points for the rectangles.

2. In this problem, you may choose values for a and b, or you may do your computations with a and b as symbolic constants.

   (a) Find a formula for the right Riemann sum of $f(x) = 2x+1$ over the interval $[a, b]$, where a and b are constants with $a < b$.

   (b) Compute the area under the graph of $y = f(x)$ as a limit. (Your answer may involve a and b.)
   You may use the following fact as given: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ (but for general interest I am attaching a proof of that fact that you can read later.)

   (c) Check your answer using geometry.

3. Prove that $\frac{1}{3} \leq \int_4^6 \frac{1}{x}\,dx \leq \frac{1}{2}$ without evaluating the integral. (Hint: Draw a picture and think about areas.)

4. Compute the following integrals by drawing a graph and using what you know about area.

   (a) $\int_1^4 x - 2\,dx$

   (b) $\int_0^3 |x-1|\,dx$

   (c) $\int_0^1 \sqrt{1-x^2}\,dx$