1. Compute the following integrals by drawing a graph and using what you know about area.

   (a) $\int_1^4 x - 2\,dx$

   (b) $\int_0^3 |x-1|\,dx$

   (c) $\int_0^1 \sqrt{1-x^2}\,dx$

2. Prove that $\frac{1}{3} \leq \int_4^6 \frac{1}{x}\,dx \leq \frac{1}{2}$ without evaluating the integral. (Hint: Draw a picture and think about areas.)

3. We know how to compute the average of n numbers: add them up and divide by n. But how can we compute the average of a continuous function? This problem will answer that question. Prof. Bob Loblaw knows that the temperature (heat) of an object varies with time as $H(t) = 3t - 10$ for $0 \leq t \leq 4$.

   (a) Explain why $\frac{1}{8}\sum_{j=0}^{7} H(j/2)$ is a reasonable approximation to the average of the temperature function over the interval$[0, 4]$.

   (b) Show that, if we approximate the average temperature with n subdivisions, Prof. Bob Loblaw can write the approximation formula as

   $$A_n = \frac{1}{b-a}\sum_{j=0}^{n} H(j/n)\frac{b-a}{n}.$$

   (c) What is the limit as $n \to \infty$ of the formula in the previous part? Use this answer to compute the average of the continuous function over $[0, 4]$.

   (d) What is a formula in general for the average value of a continuous function $f(x)$ on the interval $[a, b]$?

4. In each of the following sketch a graph(s) with the properties:

   (a) $f(x)$ is not constant 0 and $\int_{-a}^{a} f(x)\,dx = 0$.

   (b) $f(x) < 0$ for $x \in (-2, -1)$ and $\int_{-3}^{0} f(x)\,dx > 0$.

   (c) $f(x),\ g(x) < 0\ \forall\ x$ and $\int_1^2 (f(x) - g(x))\,dx > 0$.

   (d) $\int_0^{2\pi} f(x)\,dx = 0,\ \int_0^{2\pi} |f(x)|\,dx = 2\int_0^{\pi} f(x)\,dx$.