When working on the following problems, consider all the material we have learned throughout the semester. That is, these problems don't all come from one section.

1. Write the following limits as definite integrals:

   (a) $\lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\frac{b-a}{n})\frac{b-a}{n}$

   (b) $\lim_{n\to\infty}\sum_{i=1}^{n}(2+\frac{2i}{n})^2\frac{2}{n}$

   (Hint: Write out the general definition of the definite integral as a limit of Riemann sums. Remember to think about the geometry involved in this definition, don't just blindly memorize it.)

2. Find all continuous functions $f(x)$ which satisfy the equation

$$(f(x))^2 = \int_0^x f(t)\frac{t}{1+t^2}\, dt.$$

   (Hint: What operation do you know how to do to both sides of this? When you don't know how to start a problem, that's often a good place to start.)

3. Prove that $f(x) = x^3 - 3x + c$ never has two roots in $[0, 1]$ no matter what c is. (Hint: What would the graph have to look like if it did have two roots? How can you prove that the graph doesn't look like that?)

4. (a) For $0 \leq x \leq 1$, let $f(x) =$ the first digit in the decimal expansion of x; e.g., $f(.713) = 7, f(1/4) = 2$.

      i. Draw a graph of f.

      ii. Compute $\int_0^1 f(x)dx$

   (b) Let $g(x) = [x] =$ "the greatest integer" in x (the largest integer smaller than or equal to x); e.g., $[5/2] = 2, [.4] = 0,$ and $[-1.2] = -2$.

      i. Draw a graph of g .

      ii. Compute $\int_{-100}^{100} g(x)dx$

5. A giddily gleeful student, elated over passing a Calculus 180 examination, hurls a somewhat large calculus book directly upward from the ground. It moves according to the law $s(t) = 96t - 16t^2$ where t is the time in seconds after it is thrown and $s(t)$ is the height in feet above the ground at time t. Find:

   (a) the velocity of the book after 1.5 seconds;

   (b) the maximum height the book reaches;

   (c) the average speed of the book during its upward rise;

   (d) the acceleration of the book at its maximum height;

   (e) the rate of change of the acceleration of the book after 4 seconds;

   (f) the time it would take for the 6 ft. tall student to have the misfortune of being hit on the head by the book.

6. Prove that it is impossible to find two differentiable functions $f(x)$ and $g(x)$ for which $f(0) = g(0) = 0$ and which satisfy $f(x)g(x) = x$ for all x. *(Hint: differentiate)*

7. (a) Given an $\epsilon > 0$, can one always find a $\delta > 0$ such that $|1/x^2 - 1| < \epsilon$ whenever $0 < |x+1| < \delta$? Why or why not?
   (b) How can you find such a δ?
   (c) What can you prove using the above information?

8. Extra challenge problem: Let $f(x)$ be differentiable on $[0, 1]$, the second derivative exists for all $x \in [0, 1]$ and $f'(0) = 0,\ f'(1) = 0,\ f(0) = 0,$ and $f(1) = 1$.
   Must there exist an $a \in (0, 1),\ |f''(a)| \geq 4$.

Next time, we will be doing exam practice. You should come to next class having studied and be prepared to take a practice final!