

1. Intro to limits at infinity:

(a) What is $\lim_{x \rightarrow \infty} \frac{1}{x}$? (Hint: picture the graph.)

(b) What is $\lim_{x \rightarrow \infty} \frac{1}{x^n}$ where $n \geq 1$?

(c) Use the above facts and some algebra to evaluate $\lim_{x \rightarrow \infty} \frac{x^3 + 3x + 1}{5x^3 - 2x^2 + 17x - \pi}$

2. Fill in the boxes in the following definitions.

(a) The line \square is called a *vertical asymptote* of a function f if $\lim_{x \rightarrow \square} f(x) = \square$.

(b) The line \square is called a *horizontal asymptote* of a function f if $\lim_{x \rightarrow \square} f(x) = \square$ or $\lim_{x \rightarrow \square} f(x) = \square$.

3. Find the vertical and horizontal asymptotes of the following functions.

(a) $h(x) = \frac{x^2 - 9}{x(x - 3)}$

(b) $f(x) = e^{1/x}$

(c) $g(x) = \frac{\cos x + 2\sqrt{x}}{\sqrt{x}}$

4. Let $g(x) = x^{1/3}$.

(a) Write the expression for the slope of the secant line that passes through $(0, 0)$ and $(h, g(h))$.

(b) Calculate the limits of the slopes of the secant lines as h approaches 0 from the left and the right.

(c) What can you say about the line tangent to the graph of g at $(0, 0)$? Verify your answer by drawing the graph of the function.

5. Recall that a rational function is a ratio of two polynomial functions:

$$f(x) = \frac{a_m x^m + a_{m-1} x^{m-1} + \cdots + a_2 x^2 + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_2 x^2 + b_1 x + b_0}.$$

(a) Prove that if $m = n$, then $\lim_{x \rightarrow \pm\infty} f(x) = \frac{a_m}{b_n}$. (Hint: think about generalizing what you did in number 1c.)

(b) What happens if $m < n$?

(c) What happens if $m > n$?