

1. **Without** looking up the definition in your textbook, state very precisely what it means to say $\lim_{x \rightarrow a} f(x) = +\infty$ (where a is a finite number). Discuss your definition with the other members of your group, and try to make sure it is completely accurate and detailed. Draw a graph to illustrate your definition. Label relevant quantities on the x and y axes, and state your definition in terms of those. (This question should take at least several minutes of discussion.)
2. Use your definition to prove carefully that $\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$.
3. **Without** looking up the definition in your textbook, state very precisely what it means to say $\lim_{x \rightarrow a} f(x) = L$ (where a and L are each finite numbers). Discuss your definition with the other members of your group, and try to make sure it is completely accurate and detailed. Draw a graph to illustrate your definition. Label relevant quantities on the x and y axes, and state your definition in terms of those. (This question should take at least several minutes of discussion.)
4. Does your definition make sense for the case where f has a removable discontinuity at a ? If not, think about how to modify your definition so that it does.
5. Can you use your definition to prove that $\lim_{x \rightarrow 2} (3x) = 6$?

6. Let

$$f(x) = \begin{cases} 3x & : x \neq 2 \\ 5 & : x = 2 \end{cases}$$

Use your definition to prove that $\lim_{x \rightarrow 2} (3x) = 6$. Is this proof different than the previous one?

7. Let

$$f(x) = \begin{cases} 1 & : x \leq 0 \\ 2 & : x > 0 \end{cases}$$

Can you use your definition to prove that $\lim_{x \rightarrow 0} f(x)$ does not exist?

8. If you have time, try proving that $\lim_{x \rightarrow 2} (x^2) = 4$. What step is different from the above proofs? To deal with that step, think about this: is your definition of the limit a global or local property of the function? Can you add a reasonable assumption that will make this proof work? (We'll discuss this more next time, so keep thinking about it.)