

1.
 - (a) Given a function $f(x)$, write down a formula for the secant line between the points $(a, f(a))$ and $(x, f(x))$. Sketch a picture showing the secant line.
 - (b) Using a limit, write a formula for the slope of the tangent line at the point $(a, f(a))$. This is called the derivative of f at a , and is written $f'(a)$.
 - (c) We can express the same derivative in a different way. Sketch the same picture as above, but now label the distance between x and a by h . Write an expression for $f'(a)$ in terms of a and h . (Note that you can also let x play the role of a above and get an expression for $f'(x)$ for any x .)
2. Let $f(x) = x^2$.
 - (a) Using each of the above definitions, evaluate $f'(0)$ and $f'(1)$. Verify on a sketch that these slopes make sense.
 - (b) Find an expression for $f'(x)$ that is valid for all values of x in the domain of f .
3. Let $f(x) = \sqrt{x+1}$.
 - (a) Evaluate $f'(3)$
 - (b) Find an expression for $f'(x)$ that is valid for all values of x in the domain of f .
4. Derivatives of products:
 - (a) Let $f(x) = x+1$, $g(x) = x^2$. Use the definition of the derivative to calculate $f'(x)$, $g'(x)$, $[f(x)g(x)]'$, $f'(x)g'(x)$ and $f'(x)g(x) + f(x)g'(x)$. Which are equal?
 - (b) Assume f and g are functions that are differentiable at x . Use the definition of the derivative to prove that $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$. (Hint: add and subtract $f(x)g(x+h)$ in the numerator and then break your expression up into two fractions.)
5. On the back of this paper there is a sample homework problem with a proof. Suppose your friend wrote this proof and then asked you if it made sense. Read over it and critique it. Are there any mistakes? If so, explain why they are wrong, and correct them. (Remember, the point is not just to make sure the final answer makes sense, but to check that each step makes sense logically and is expressed clearly.)
6. Let
$$f(x) = \begin{cases} -x & : x \leq 0 \\ x^2 & : x > 0 \end{cases}$$
 - (a) Is $f(x)$ continuous at $x = 0$?
 - (b) Is $f'(0)$ defined? If so, what is it?