

1. Determine whether the following statements are true or false, giving an explanation or a counterexample.

- (a) The slope of the tangent line to  $f(x) = e^x$  is never zero.  
(b)  $f(x) = e^x$  is the only function such that  $f'(x) = f(x)$  for all  $x$ .  
(c)  $\frac{d}{dx}e^x = xe^{x-1}$   
(d) The  $n$ th derivative  $\frac{d^n}{dx^n}x^3 + 2x = 9$  equals 0 for any integer  $n \geq 3$ .  
(e)  $\frac{d}{dx}\left(\frac{x^2 - 4}{x + 2}\right) = \frac{2x}{1} = 2x$

2. Find the equation of the tangent line to the given function at  $x = a$ .

- (a)  $y = 2e^x - 4x^2 + 3x$ ;  $a = 0$   
(b)  $f(x) = 4x^2 - 3x + 1$ ;  $a = 2$

3. Let

$$f(x) = \begin{cases} x^2 - 2 & x \geq 2; \\ 3x - 4 & x < 2. \end{cases}$$

- (a) Is  $f$  continuous?  
(b) Find a formula giving the slope of the secant line intersecting  $f(x)$  when  $x = 2$  and  $x = 2 + h$ . You should have different answers for  $h < 0$  and  $h > 0$ . Why?  
(c) Find the limit of these slopes as  $h \rightarrow 0^-$  and  $h \rightarrow 0^+$ .  
(d) Is  $f$  differentiable?
4. Do each problem in two ways: use the product or quotient rule, and then rewrite the function and apply the power rule.
- (a)  $f(t) = (2t + 1)(t^2 - 2)$   
(b)  $h(t) = \frac{t^2 - 1}{t - 1}$

5. Derive the derivatives of  $\sec x$ ,  $\csc x$ ,  $\tan x$ , and  $\cot x$  given

$$\frac{d}{dx}\sin x = \cos x \quad \frac{d}{dx}\cos x = -\sin x.$$

6. Use the limit definition to show that if  $f(x)$  is differentiable and  $f(x) \neq 0$ , then  $1/f(x)$  is differentiable and

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{f^2(x)}$$

Can you use this fact plus the product rule to prove the full quotient rule?