

Evaluate the following limits. Show your work. If you use l'Hospital's rule, be sure to check that it applies and indicate where you used it.

1. $\lim_{x \rightarrow \pi/4} \frac{\sin(x) - \cos(x)}{\tan(x)}$

2. $\lim_{x \rightarrow 1} \frac{\ln(x) + x - 1}{\sin(\pi x)}$

$$\begin{aligned} \boxed{1} \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{\tan(x)} &= \frac{\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} \\ &= \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{1} = \boxed{0} \end{aligned}$$

$$\boxed{2} \quad \lim_{x \rightarrow 1} \frac{\ln(x) + x - 1}{\sin(\pi x)} \quad \text{plugging in gives} \quad \frac{0 + 1 - 1}{0} = \frac{0}{0} \quad \text{indeterminate form}$$

So by l'Hôpital's Rule

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{\frac{1}{x} + 1}{\pi \cos(\pi x)} = \frac{\frac{1}{1} + 1}{\pi \cos(\pi)} = \frac{2}{\pi(-1)} = \boxed{\frac{-2}{\pi}}$$