

1. Find a constant c so that $\lim_{x \rightarrow -1} f(x)$ exists. Justify your answer.

$$f(x) = \begin{cases} \frac{cx^2 - c}{x+1} & : x > -1 \\ cx + 3 & : x \leq -1 \end{cases}$$

$\lim_{x \rightarrow -1} f(x)$ exists if $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^-} f(x)$

$$\begin{aligned} \lim_{x \rightarrow -1^+} f(x) &= \lim_{x \rightarrow -1^+} \frac{cx^2 - c}{x+1} = \lim_{x \rightarrow -1^+} \frac{c(x+1)(x-1)}{x+1} = c(-1-1) \\ &= -2c \end{aligned}$$

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} cx + 3 = -c + 3$$

$$\text{Set } -2c = -c + 3$$

$$\Rightarrow -c = 3$$

$$\Rightarrow \boxed{c = -3}$$