Name: Solutions Quiz 3 Circle: 9 AM, 10 AM, 11 AM

1. Use the Intermediate Value Theorem to prove that the equation $x - 2 = -x^{\frac{1}{3}}$ has at least one (real) solution. (You should (briefly) verify that the IVT applies and state where you use it.)

$$x - 2 = -\sqrt[3]{x} \iff x + \sqrt[3]{x} - 2 = 0$$

Let $f(x) = x + \sqrt[3]{x} - 2$

Note $f(x)$ is continuous on the whole real line, since x, $\sqrt[3]{x}$, and 2 are cts functions on all real numbers.

$f(27) = 27 + 3 - 2 > 0$

$f(-27) = -27 - 3 - 2 < 0$

So by the IVT, there is an x_0 in $(-27, 27)$ such that $f(x_0) = 0$

ie, such that x_0 is a solution to the original equation.

[Note: This is one correct solution, but not the only possible one.]

More comments:

- By definition, a polynomial in x is a function of the form $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$. Notice there are only positive whole number powers of x. So $\sqrt[3]{x}$ is not a polynomial, but it is a continuous function on the whole real line.

- The solution x_0 that you get is between the x-values you chose, not the y-values. So in my solution, I get $-27 < x_0 < 27$, not $-32 < x_0 < 28$.

- $f(x)$ is not given. So before you say anything like $f(0)$, you should say what you're defining $f(x)$ to be.