

1. Consider the equation $x^4 - x^2y + y^4 = 1$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

(b) Find the slope of the tangent line at the point $(-1,1)$. (Simplify your answer).

$$\boxed{a} \quad x^4 - x^2y + y^4 = 1$$

$$\Rightarrow \frac{d}{dx} (4x^3 - (2xy + x^2 \frac{dy}{dx}) + 4y^3 \cdot \frac{dy}{dx}) = 0$$

$$4x^3 - 2xy - x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 0$$

$$4x^3 - 2xy = x^2 \frac{dy}{dx} - 4y^3 \frac{dy}{dx}$$
$$= \frac{dy}{dx} (x^2 - 4y^3)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{4x^3 - 2xy}{x^2 - 4y^3}}$$

$$\boxed{b} \quad \left. \frac{dy}{dx} \right|_{(-1,1)} = \frac{4(-1)^3 - 2(-1)(1)}{(-1)^2 - 4(1)^3} = \frac{-4 + 2}{1 - 4} = \frac{-2}{-3} = \boxed{\frac{2}{3}}$$