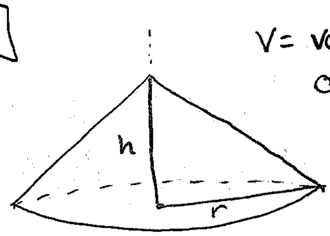
Name: Solutions Quiz 9 Circle: 9 AM, 10 AM, 11 AM

Solve the following problems. In order to get credit, you must use calculus to prove that your answer is correct (not just geometrical or physical intuition).

1. Salt falls from a funnel, piling up in a conical pile with a radius that is always four times its height. If the salt falls from the funnel at a constant rate of twelve cubic inches per minute, how fast is the height of the pile changing at the moment when the pile is two inches high? (Hint: recall that a cone with height h, lateral side length l and base a circle of radius r has volume $\frac{1}{3}\pi r^2 h$ and lateral surface area $\pi r l$. Note this does not mean you need both formulas - you need to decide which, if any, are relevant to the problem.)

2. A farmer builds a rectangular pen with one side up against a barn (so on that side, no fence is needed). There are 500 feet of fencing material available. Determine the dimensions of the pen that will enclose the largest area.

$\boxed{\#1}$



V= volume of cone

12 cubic inches per min is the rate of change of volume.

$$\frac{dV}{dt} = 12 \text{ in}^3/\text{min}$$

$$r = 4h$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (4h)^2 h = \frac{1}{3}\pi 16 h^3$$

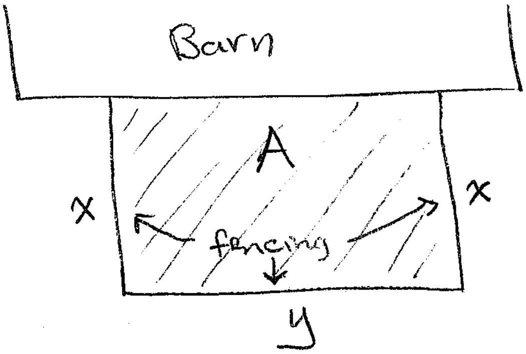
$$\overset{d/dt}{\Rightarrow} \quad \frac{dV}{dt} = \frac{1}{\cancel{3}}\pi 16 \cdot \cancel{3} h^2 \frac{dh}{dt} = 16\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 12$$

When $h = 2$: $\quad 12 = 16\pi (2)^2 \frac{dh}{dt}$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{12}{16\pi \cdot 4} = \boxed{\frac{3}{16\pi} \text{ in}^2/\text{min}}$$

#2



$2x + y = 500$

$\Rightarrow y = 500 - 2x$

Area $= A = xy$

$A(x) = x(500-2x) = 500x - 2x^2$

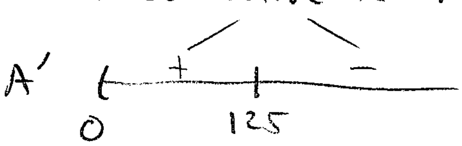
$A'(x) = 500 - 4x \overset{\text{set}}{=} 0$

$\Rightarrow 500 = 4x$

$\frac{500}{4} = 125 = x$ So $x = 125$ is the only critical pt of A.

Check $x=125$ gives a maximum of A:

1st derivative test:



(because Area of a real pen can't be ≤ 0.)

or

2nd derivative test:

$A''(x) = -4$

$\Rightarrow$ A is concave down everywhere, and thus at $x = 125$

So, A has a maximum at $x = 125$

$x = 125 \Rightarrow y = 500 - 2(125) = 500 - 250 = 250$

So $\boxed{x = 125 \text{ ft}, \ y = 250 \text{ ft are the dimensions that enclose the maximum area.}}$