Math 180 – Fall 2012 Name: Solutions

Quiz 6, Oct 18th, 2012

Write your answers on this page. Continue on the back if you need more space.

(5 pts) **1.** Using logarithmic differentiation find the slope of the line tangent to $y = x^{\sin x}$ at the point $x = 1$.

Method 1: $x^{\sin x} = e^{\ln(x^{\sin x})}$

$= e^{\sin x \cdot \ln(x)}$

$\frac{d}{dx}\left(e^{\sin x \cdot \ln(x)}\right) = e^{\sin x \cdot \ln x}\left[\cos x \cdot \ln x + \frac{1}{x}\sin x\right]$

$= x^{\sin x}\left[\cos x \cdot \ln x + \frac{1}{x}\sin x\right]$

Method 2: $\ln(y) = \ln(x^{\sin x})$

$\ln y = \sin x \cdot \ln(x)$

$d/dx \Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \cos x \cdot \ln x + \frac{1}{x}\sin x$

$\frac{dy}{dx} = y\cdot\left[\cos x \cdot \ln x + \frac{1}{x}\sin x\right]$

$= x^{\sin x}\left[\cos x \cdot \ln x + \frac{1}{x}\sin x\right]$

Then Evaluate at $x=1$

$\left.\frac{dy}{dx}\right|_{x=1} = \cancel{1}^{\sin(1)}\left[\cos(1)\cdot \cancel{\ln(1)}^{0} + \frac{1}{1}\cdot\sin(1)\right] = \boxed{\sin(1)}$

(no easy simplification here)

(5 pts) **2.** Find $(f^{-1})'(5)$ if $f(x) = x^2 + 1$, restricted to $x > 0$. (added)

$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))}$

$= \frac{1}{f'(2)}$

$= \frac{1}{2\cdot 2} = \boxed{\frac{1}{4}}$

Set $x^2+1 = 5$

$x^2 = 4$

$\Rightarrow x = \pm 2$ take $+2$ since $f(x)$ is defined on $x>0$.

So $f^{-1}(5) = 2$

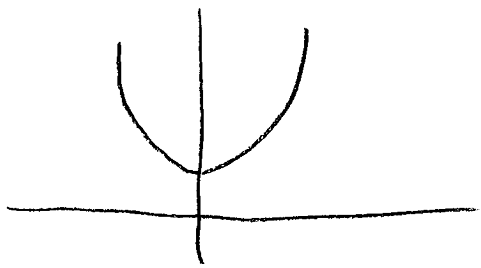
$f'(x) = 2x$

Note: The restriction $x>0$ is needed because $f(x) = x^2+1$ on the whole real line is not invertible – its "inverse" would fail the vertical line test. In order to be invertible, $f(x)$ must pass the horizontal line test (as well as the vertical line test to be a function).

Note also: $f^{-1}(x)$ here, if defined at $x=0$, would have an undefined slope at $x=0$, but that's ok – it means a vertical tangent line here.

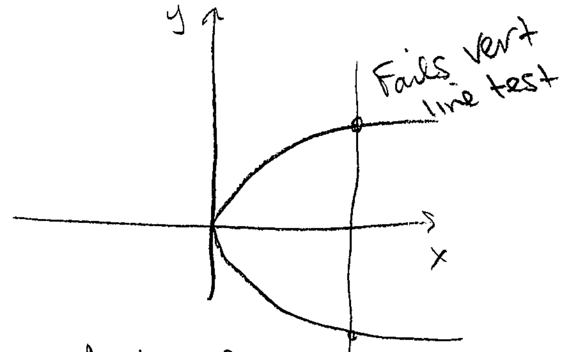
More detail

$f(x) = x^2+1$ on all the line:



Not invertible.

reflect around $y=x$ $\Rightarrow$

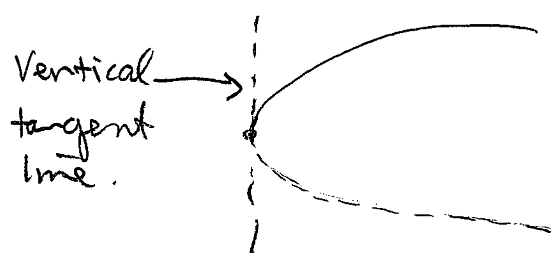


Not a function

Need to restrict to some domain where $f(x)$ passes horiz line test - $x>0$ works, so would $x>10$, or $x<-5$, etc. Then $f(x)$ passes horiz line test, so $f^{-1}(x)$ passes vert line test (exists as a function).

At $x=0$, we have $f'(0)=0$, $(f^{-1})'(0) = \frac{1}{0}$ undefined, which doesn't mean f^{-1} doesn't exist. It means $f^{-1}(x)$ has a vert. tangent line at $x=0$.

So if we include $x=0$ in the domain, $f^{-1}(x)$ is defined, but is not differentiable at $x=0$.



We can still describe its slope at that pt.