Name: Solutions Quiz 3

1. Find a constant c so that $lim_{x\to 1} f(x)$ exists. Justify your answer.

$$f(x) = \begin{cases} \dfrac{cx^3 - cx}{x^2 - x} & : x > 1 \\ cx^4 + 5 & : x \leq 1 \end{cases}$$

2. Use the Intermediate Value Theorem to prove that the function $f(x) = x - 1 + x^{\frac{1}{4}}$ has at least one (real) root. (You should (briefly) verify that the IVT applies and state where you use it.)

$\boxed{\#1}$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} \frac{cx^3 - cx}{x^2 - x} = \lim_{x\to 1} \frac{cx(x^2-1)}{x(x-1)}$$

$$= \lim_{x\to 1} \frac{c\cancel{x}\cancel{(x-1)}(x+1)}{\cancel{x}\cancel{(x-1)}} = \lim_{x\to 1} c(x+1) = c(2) = 2c$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} c(x^4) + 5 = c(1)^4 + 5 = c + 5$$

Set $2c = c + 5$
$\quad -c \quad -c$ $\Rightarrow \boxed{c = 5}$

$\boxed{\# 2}$ $f(x)$ is continuous for $x \geq 0$ ($x^{1/4} = \sqrt[4]{x}$ is not defined for $x < 0$) so the IVT applies for $x \geq 0$.

$f(0) = -1 < 0$

$f(1) = 1 - 1 + 1^{1/4} = 1 - 1 + 1 = 1 > 0$

So by the IVT, $f(x)$ has at least one (real) root between $x = 0$ and $x = 1$.

Note: $\sqrt[4]{x}$ is not a "polynomial"! a polynomial has only positive integer powers of x.