$\boxed{\begin{array}{l} 2.7 \ \#20 \\ \text{Solution} \end{array}}$ Prove that $\lim_{x \to 3} \overbrace{(-2x+8)}^{f(x)} = 2$

Proof: Let $\varepsilon > 0$. Let $|x-3| < \delta = \underline{\varepsilon/2}$

Then $|-2x+8-2| = |-2x+6| = |-2||x-3|$

$= 2\underbrace{|x-3|}_{<\delta} = 2 \cdot \delta = 2 \cdot (\varepsilon/2) = \varepsilon$ ✓

So for every $\underline{\varepsilon > 0}$ we found a $\delta = \frac{\varepsilon}{2}$ $(\underline{\delta > 0})$

so that whenever $|x-3| < \delta$, then $|f(x)-2| < \varepsilon$.

So by the definition of the limit, $\lim_{x \to 3} (-2x+8) = 2$. ∎

Note: ε, δ both must be positive. They are distances: