Name: Math 180 Final Exam Practice Problems

Justify your solutions and show your work.

1. Using the precise definition of the limit, prove that $\lim_{x \to 2} 3x + 1 = 7$.

2. Let $f(x) = x^5 + 3x^3 + 2$.
   (a) Prove that $f(x)$ has at least one real root.
   (b) Prove that $f(x)$ has *only* one real root.

3. Let $f(x) = \frac{x}{e^x}$.
   (a) Find all x-intercepts of $f(x)$.
   (b) Find the intervals of monotonicity and all local extrema of $f(x)$.
   (c) Find the intervals of concavity and all inflection points of $f(x)$.
   (d) Find all vertical asymptotes of $f(x)$.
   (e) Find all horizontal asymptotes of $f(x)$.
   (f) Draw a sketch of $f(x)$ based on the above information.

4. Compute the following limits.
   (a) $\lim_{x \to \pi} (1 - \sin x)^{\cos x}$
   (b) $\lim_{x \to 0+} \frac{e^{2x} - 1}{e^{3x} - 1}$
   (c) $\lim_{x \to \infty} \frac{x^3 + \sqrt{x^6 + 7} + 3x + 5}{3x^3 - 2x + 7}$
   (d) $\lim_{x \to 1} \frac{(ln(x) + x - 1)}{sin(\pi x)}$
   (e) $\lim_{x \to 1} (ln(x) + x - 1)^{sin(\pi x)}$

5. Evaluate the following:
   (a) $\int \frac{\sin(\ln x)}{x} \, dx$
   (b) $\int_{-5}^{5} x^2 sin(5x) dx.$
   (c) $\int_{-2}^{0} 2x^2 \sqrt{1 - 4x^3} dx.$

6. $\int_0^1 x^2 dx$ is equal to which of the following?
   (a) $\lim_{n \to \infty} \sum_{k=1}^{n} (1/n)(k/n)^3$
   (b) $\lim_{n \to \infty} \sum_{k=1}^{n} (1/n)(k/n)^2$
   (c) $\lim_{n \to \infty} \sum_{k=1}^{n} (5/n)(k/n)^2$

(d) $\lim_{n\to 0} \sum_{k=1}^{n} (k/n)^2$

7. Air is being pumped into a spherical balloon at a rate of 5 cm3/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

8. Let $f(x) = x^3 + 2x$. Find $(f^{-1})'(3)$.

9. An open rectangular box with a square base ("open" means it has no top) needs to be made out of 48 ft^2 of material. find the dimensions that give the largest possible volume.

10. Let $f(x) = \int_3^x \frac{1}{1+t^3} dt$. Find the best linear approximation to $f(x)$ at x=3.

11. Find the average value of $f(x) = 6x^2 - 5x + 2$ on the interval $[-3, 1]$.