Justify your solutions and show your work.

1. Differentiate the following functions (with respect to x).

   (a) $f(x) = x^3 \cos(e^{x^2 + 2x})$

   (b) $g(x) = \dfrac{2^{3x+1}}{\ln x}$

   (c) $h(x) = \displaystyle\int_3^{2x} \dfrac{1}{1+t^3}\, dt$

2. Sketch a graph of the function $f(x) = \frac{x}{x^2+1}$. Label with x and y coordinates all local maxima and minima and inflection points. Also indicate any asymptotes or holes.

3. Compute the following limits.

   (a) $\lim_{x \to \pi}(1 - \sin x)^{\cos x}$

   (b) $\displaystyle\lim_{x \to 0+} \dfrac{e^{2x} - 1}{e^{3x} - 1}$

   (c) $\displaystyle\lim_{x \to \infty} \dfrac{x^3 + 3x + 5}{3x^3 - 2x + 7}$

4. Ted wants to build a box with a square base and surface area 5 sq. meters. What is the largest possible volume that his box can have? (Use calculus to justify your answer.)

5. Evaluate the following:

   (a) $\displaystyle\int_0^2 u\sqrt{4u + 3}\, du$.

   (b) $\displaystyle\int \dfrac{\sin(\ln x)}{x}\, dx$

6. The average value of a function f over an interval $[a, b]$ is given by $\frac{1}{b-a}\int_a^b f(x)\, dx$. find the average value of $f(x) = x(x^2 - 7)^3$ over the interval $[-1, 1]$.