Justify your solutions and show your work.

1. The graph of $f^{\prime}$ on the interval $[-3,2]$ is shown in the figure.

(a) On what inteval(s) is $f$ increasing? Decreasing?
(b) Find the critical points of $f$. Identify whether the critical points correspond to minima, maxima, or neither for $f$.
(c) On what intervals is $f$ concave up? Concave down?
(d) Sketch the graph of $f^{\prime \prime}$.
(e) Sketch a possible graph of $f$.
2. Consider the curve defined by $y^{2} \sin x+\cos (x y)+y^{2}=2$. Find the tangent to this curve at the point $(0,1)$.
3. Find the average value of $f(x)=x\left(x^{2}-7\right)^{3}$ over the interval $[-1,1]$.
4. Find the point(s) on the graph of $y=x^{2}+1$ that is/are closest to the point $(0,2)$.
5. Suppose that a function $f(x)$ is defined and has $f^{\prime}(x)<0$ and $f^{\prime \prime}(x)<0$ for all $x$. Suppose also $f(3)=5$ and $f^{\prime}(3)=-2$. (a) Find an integer $n$ such that $|f(2)-n|<1$. (b) Suppose $f(r)=0$ and find an integer $k$ such that $|r-k|<2$.
