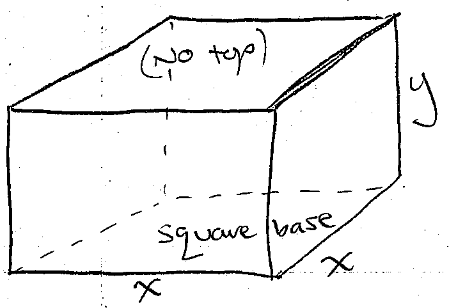
Name: Solutions Quiz 10

1. An open rectangular box with a square base needs to be made out of 48 ft^2 of material. Find the dimensions that give the largest possible volume. (To get full credit you must use calculus to show your work is correct, and don't forget to verify that your answer actually does give a maximum.)



$Vol = V = x^2 \cdot y$

Surface Area = $48\ ft^2$

$SA = x^2 + 4xy = 48$

$\Rightarrow y = \dfrac{48 - x^2}{4x}$

$V(x) = x^2 \cdot \left(\dfrac{48 - x^2}{4x}\right) = 12x - \frac{1}{4}x^3$ (domain $x > 0$ since length of side)

$V'(x) = 12 - \frac{3}{4} \cdot x^2 \stackrel{set}{=} 0$

$12 = \frac{3}{4}x^2$

$x^2 = \frac{4}{3} \cdot 12 = 4 \cdot 4 = 16 \Rightarrow x = 4$ is our only crit pt. ($x \neq -4$ since physical distance)

Check max:

$V''(x) = -\frac{6}{4}x \Rightarrow V''(4) = -6 < 0$

So 2nd derivative test $\Rightarrow$ $x=4$ gives a max of $V(x)$

$x = 4 \Rightarrow y = \dfrac{48 - (4)^2}{4 \cdot 4} = \dfrac{48 - 16}{16} = \dfrac{32}{16} = 2$

Dimensions for largest volume: $x = 4$ ft (base side)
$y = 2$ ft (height)

$\Rightarrow$ Max volume is $(4)^2 \cdot 2 = 16 \cdot 2 = 32\ ft^3$