Name: Solutions Quiz 2

1. Find a constant $c$ so that $\lim_{x\to -2} f(x)$ exists. Justify your answer. (To get full credit, you should show your work in terms of limits.)

$$f(x) = \begin{cases} \dfrac{cx^2 - 4c}{x+2} & : x > -2 \\ cx + \pi & : x \le -2 \end{cases}$$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \dfrac{cx^2-4c}{x+2} = \lim_{x\to -2^+} \dfrac{c(x-2)(x+2)}{x+2}$

$= \lim_{x\to -2^+} c(x-2) = c(-2-2) = -4c$

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} cx+\pi = -2c+\pi$

For $\lim_{x\to -2} f(x)$ to exist, we need the left and right hand limits to be equal.

Set $-4c = -2c+\pi$

$-2c = \pi$

$\boxed{c = -\frac{\pi}{2}}$

Note: In general, we do not need $f(-2) = \lim_{x\to -2} f(x)$ for the limit to exist — we need that for continuity, which is not what this problem asks.