Name: Solutions Quiz 4

1. (a) Let $f(x) = 3x^2$. Use the definition of the derivative to find $f'(x)$. (You will not get credit here for using shortcut rules, though you can use them to check your answer.) (4 pts)

   (b) Find $f'(5)$. (1 pt).

2. Let $f(x) = 4x + 1$. Use the formal (ϵ-δ) definition of the limit to prove that $\lim_{x\to 2} f(x) = 9$. (5 pts)

$\boxed{\#1}$ (a) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$

$= \lim_{h\to 0} \frac{1}{h}(\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2})$

$= \lim_{h\to 0} \frac{1}{\cancel{h}}(\cancel{h}(6x + 3h)) = 6x + 3(0) = \boxed{6x}$

(b) $f'(5) = 6 \cdot 5 = 30$

$\boxed{\#2}$ Def: $\lim_{x \to a} f(x) = L \iff$ For every $\varepsilon > 0$ there is $\delta > 0$ s.t. if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$,

Here $f(x) = 4x+1$

$a = 2$, $L = 9$

$\underline{\text{Pf}}$: Let $\varepsilon > 0$ be arbitrary.

Let $\delta = \underline{\varepsilon/4}$

assume $0 < |x-a| < \delta$

Then $|f(x) - L| = |4x+1 - 9| = |4x-8|$

$= 4|x-2| < 4\delta = 4 \cdot \left(\frac{\varepsilon}{4}\right) = \varepsilon$ ✓

So $\delta = \varepsilon/4$ works.

Thus $\lim_{x \to 2} 4x+1 = 9$. ∎

Note: The key idea is that if you assume $|x-2| < \frac{\varepsilon}{4}$, then you can show $|f(x) - 9| < \varepsilon$.

Many people come up with $\delta = \frac{\varepsilon}{4}$ but did not convey that idea.