

1. Consider the equation $2x^3 + \tan(y) + xy^2 = 1$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$. (6 pts)

(b) Verify that the point $(0, \pi/4)$ satisfies the equation. (1 pt)

(c) Find the slope of the tangent line at the point $(0, \pi/4)$. (3 pts)

$$(a) \quad 2x^3 + \tan(y) + xy^2 = 1$$

$$\begin{aligned} \frac{d}{dx} \Rightarrow 4x + \sec^2(y) \cdot y' + y^2 + x2yy' &= 0 \end{aligned}$$

$$y' = \frac{-y^2 - 4x}{\sec^2(y) + 2xy}$$

$$(b) \quad 2(0)^3 + \tan\left(\frac{\pi}{4}\right) + 0 \cdot \left(\frac{\pi}{4}\right)^2 = \tan\left(\frac{\pi}{4}\right) = 1 \quad \checkmark$$

$$\begin{aligned} (c) \quad y' \Big|_{(0, \frac{\pi}{4})} &= \frac{-\left(\frac{\pi}{4}\right)^2 - 4 \cdot 0}{\left(\sec\left(\frac{\pi}{4}\right)\right)^2 + 0} \\ &= \frac{-\left(\frac{\pi}{4}\right)^2}{\left(\frac{2}{\sqrt{2}}\right)^2} \end{aligned}$$