Sketch a graph of $f(x) = \dfrac{x}{x^2+1}$

Domain: all real #s

$x^2+1 = 0 \Rightarrow$ no sol'n

$\Rightarrow$ **no vert asymptotes**

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} \overset{\text{set}}{=} 0 \Rightarrow$$

$x = \pm 1$ crit pts

$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)\left(2(x^2+1)(2x)\right)}{(x^2+1)^4} = \frac{(x^2+1)(2x)\left[-(x^2+1) - 2(1-x^2)\right]}{(x^2+1)^4}$$

$$= \frac{(2x)\left[-x^2-1-2+2x^2\right]}{(x^2+1)^3} = \frac{2x\left[x^2-3\right]}{(x^2+1)^3}$$

Set $f(x) = 0$

$\Rightarrow$ **$x = 0$ is the only x-intercept**

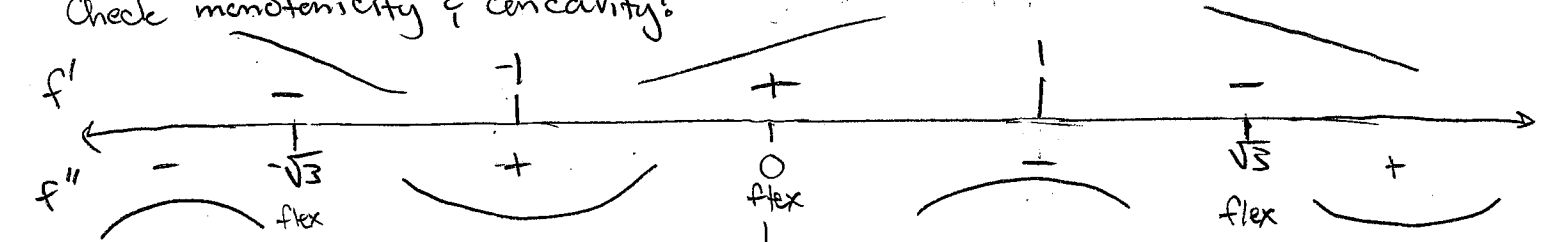
$$= \frac{2x\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}{(x^2+1)^3} \overset{\text{set}}{=} 0 \Rightarrow$$

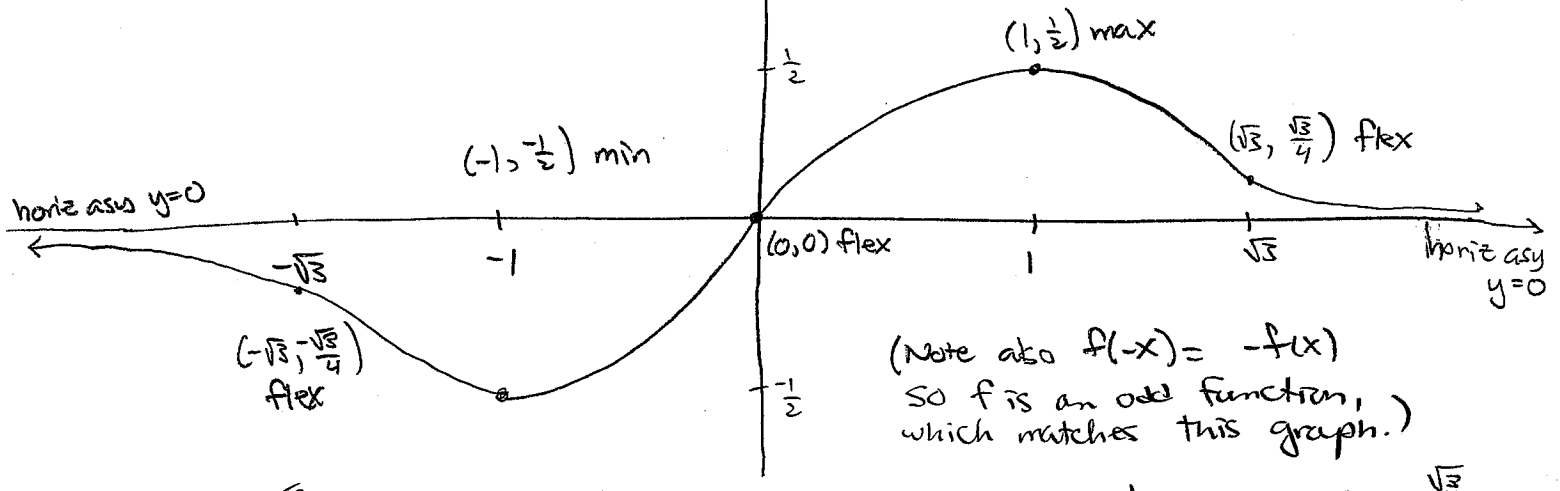
$x = 0, \pm\sqrt{3}$ possible inflection pts

$\lim_{x \to \pm\infty} \dfrac{x}{x^2+1} = 0$ (since denom is higher degree)

$\Rightarrow$ **horiz asymptote (left and right) at $y = 0$**

Check monotonicity & concavity:





(Note also $f(-x) = -f(x)$ so f is an odd function, which matches this graph.)

$f(-\sqrt{3}) = -\frac{\sqrt{3}}{4}$ $\quad f(-1) = -\frac{1}{2}$ $\quad f(0) = 0$ $\quad f(1) = \frac{1}{2}$ $\quad f(\sqrt{3}) = \frac{\sqrt{3}}{4}$