Calculus II ESP

Worksheet 3: Volumes by Slicing

Name:

Recall that to define the definite integral, we approximated the area of a region under a curve by rectangles, then took the limit of the sum of those rectangles:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Similarly, if we can slice a solid object into consistent pieces, we can approximate the volume of the solid by summing the volumes of slices and then taking a limit to approach the exact volume. If we know that a solid has cross-sectional area $A(x)$ as we range from $x = a$ to $x = b$, we can write that the volume of that solid is

$$V = \lim_{n\to\infty} \sum_{k=1}^{n} A(x_k)\Delta x = \int_a^b A(x)dx$$

1. This is great if we know the cross-sectional area. Given a solid with cross sections perpendicular to the x-axis with area given by $A(x) = x^2$ and x ranging from 0 to 10, find the volume of this solid.

In the long run, most of the work goes into finding a formula for the cross-sectional area. One nice class of examples are volumes of rotation, because then the cross sectional area always involves circles. For problems 2-4, sketch the region and a representative rectangle. Also sketch the solid and the representative disc or washer you get by rotating the representative rectangle. Then, find the volume of the solid.

2. Find the volume of the solid generated by revolving the region bounded by $y = 2x$, $y = 0$, and $x = 3$ about the x axis. First use calculus, and then check your work by using the appropriate volume formula from high school geometry.
3. (a) Calculate the volume of the solid obtained by rotating the region under the curve $y = 8 - x^3$ for $0 \leq x \leq 2$ about the x axis.
   (b) Compute the volume if the same region is rotatated about the y axis.
4. If the circle $(x - b)^2 + y^2 = a^2$ $(0 < a < b)$ is revolved about the y axis, it generates doughnut-shaped solid called a torus. Set up an integral for the volume of this torus. Verify that this integral also represents the area of the circle times the distance the circle's center travels around the y axis. Find the volume of the torus (note you are not required to find the antiderivative if you see an easier way to evaluate the integral).
5. An object has the semicircle $y = \sqrt{9 - x^2}$ where $-3 \leq x \leq 3$ as its base. The cross sections perpendicular to the x-axis are squares. Find the volume of the object.