

1. (Warm-up) The following integrals represent the volume of either a hemisphere or cone. For each part, decide which shape the volume represents, and give the radius of the hemisphere, or the radius and height of the cone. Make a sketch with labels.

(a) $\int_0^6 \pi \left(3 - \frac{y}{2}\right)^2 dy.$

(b) $\int_0^{12} \pi(144 - h^2) dh.$

2. A wedge is cut from the base of a cylinder of radius 9 by a plane passing through a diameter of the base and inclined at an angle of 45° to the base.

So that we can all have consistent notation, set up your cylinder so that its base is a circle in the xy -plane centered at $(0,0)$ and have the slicing plane pass through the diameter that lies on the y -axis.

- (a) Set up an integral that represents the volume of this wedge by using cross-sections perpendicular to the y -axis.
- (b) Set up an integral that represents the volume of this wedge by using cross-sections perpendicular to the x -axis.
- (c) Evaluate each integral to verify that both of these integrals give the same answer.
3. A hollow cone is standing on its base. The base has a radius of 3 feet and the cone's height is 10 feet. What volume of water would be needed to fill the cone to a height of 6 feet?
4. A hemispherical tank is lying on its flat side. It has a radius of 2 meters. How much water is need to fill the tank to a height of 1.5 meters?