Calculus II ESP

Worksheet 8: Trig Integrals

Tues, Feb 12

Spring 2013

1. This problem is to be turned in. Write up your solution nicely and hand it in when you are done. Read it over to be sure each step makes sense and is clearly explained. Prove that $\int \sec x \; dx = \ln|\sec x + \tan x| + C$ by using the non-obvious substitution $u = \sec x + \tan x$. Can you figure out a similar trick for finding the antiderivative of the cosecant function?

2. Evaluate the following integrals.

   (a) $\displaystyle\int \sin^5 t \; dt$ (b) $\displaystyle\int \sin^2 x \cos^4 x \; dx$ (c) $\displaystyle\int \sec^5 \theta \; d\theta$

   (d) $\displaystyle\int_0^{\pi/4} \tan^3 h \sec^2 h \; dh$ (e) $\displaystyle\int_0^{\pi/8} \sqrt{1-\cos(8x)} \; dx$ (f) $\displaystyle\int_{\pi/6}^{\pi/3} \cot^3 \theta \; d\theta$

3. Household electricity is supplied in the form of alternating current that varies from 155 V to -155 V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the equation $E(t) = 155 sin(120\pi t)$, where t is the time in seconds. Voltmeters read the RMS (root-square-mean) voltage, which is the square root of the average value of $(E(t))^2$ over one cycle.

   (a) Calculate the RMS voltage of household current. (Hint: how do we generally find the average value of a function $f(x)$ on an interval $[a, b]$?)

   (b) Many electric stoves require an RMS voltage of 220 V. Find the corresponding amplitude A needed for the voltage $E(t) = A sin(120\pi t)$.

4. In this problem, we will prove the reduction formula for the sine function.

   (a) Use integration by parts to show that

   $$\int \sin^n x \; dx = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} \cos^2 x \; dx$$

   (b) Now use a trig identity to get the reduction formula:

   $$\int \sin^n x \; dx = -\frac{\sin^{n-1} \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \; dx$$