Calculus II ESP **Spring 2013**

Trig Substitution

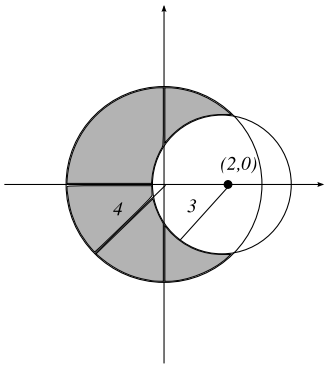
Worksheet # 9

1. **Warmup**: Take a few minutes to set up a trig substitution for this integral, and list any identities you need to use and steps you need to take to complete the problem. Then, we will go through it together on the board.

$$\int \frac{dx}{\sqrt{x^2 - 64}}$$

Group Presentations: We will break up into **three groups**. For each of problems 2,3, and 4, we will take about 15 minutes for you to work on the problem with your group. Then, I will choose a group to **go to the board and present their work**. Even if you have not finished the whole problem, you can present what you have attempted and we can finish it together on the board. Everyone in each group should be prepared to participate. I expect the rest of the class to be a constructive audience during these times: you should listen to the presenters, and you should raise your hand if you have a comment or question.

2. A lune is a crescent-shaped region bounded by the arcs of two circles. Let C_1 be a circle of radius 4 centered at the origin. Let C_2 be a circle of radius 3 centered at the point $(2, 0)$. Find the area of the lune (shaded in the figure) that lies inside C_1 and outside C_2.

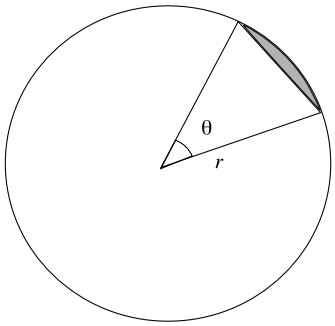


3. Evaluate the following integral using a trig substitution: $\int e^{4x}\sqrt{1 + e^{2x}}dx$

4. The area of a cap of a circle of radius r subtended by an angle θ (see figure) is given by

$$A_{\text{seg}} = \frac{1}{2}r^2(\theta - \sin\theta).$$

Verify the formula with calculus. (Bonus: verify the formula using geometry.)



5. **Cooldown**: Determine whether the following statements are true or false and give an explanation. (These should be quick. If you find yourself doing a lot of work, take a step back and see if you can find a more direct way to answer the stated question. Spend a couple minutes thinking about these and discussing them with your group, and then I will call on people to explain their answers.)

   (a) If $x = 4\tan\theta$, then $\csc\theta = 4/x$.

   (b) The integral $\int_1^2 \sqrt{1-x^2}\,dx$ does not have a finite real value.

   (c) The integral $\int_1^2 \sqrt{x^2-1}\,dx$ does not have a finite real value.

   (d) The integral $\int \frac{dx}{x^2+4x+9}$ cannot be evaluated using a trigonometric substitution.