1. Consider the curve $y = \ln x$. Find the length of the curve from $x = 1$ to $x = a$. (Hint: the change of variables $u = \sqrt{x^2 + 1}$ allows evaluation by partial fractions.)

2. Evaluate the following itegral, which needs a change in variables to make a rational integrand.
$$\int \frac{\sec \theta}{1 + \sin \theta} \, d\theta$$

3. This is a cool trick that just might win you an integration contest someday. Some integrands involving trigonometric functions can be converted into a rational integrand using the substitution $u = \tan(x/2)$ or $x = 2 \tan^{-1} u$.

   (a) First, verify that with the given substitution we have
   $$dx = \frac{2}{1 + u^2} \qquad \sin x = \frac{2u}{1 + u^2} \qquad \cos x = \frac{1 - u^2}{1 + u^2}$$

   (b) Now evaluate the following integrals with this technique.
   $$\text{i. } \int \frac{dx}{1 - \cos x} \qquad \text{ii. } \int \frac{dx}{1 + \sin x + \cos x}$$