Calculus II ESP | **Spring 2013**

Example: Gabriel's Horn

Worksheet # 13

1. Gabriel's Horn is a famous example of a shape with some interesting properties that we will explore today.

   (a) We define Gabriel's Horn as the surface obtained by taking the graph $y = \dfrac{1}{x}$ on the interval $[1, \infty)$ and rotating it about the x-axis. Make a sketch of this shape.

   (b) Make a prediction: what do you think the volume and surface area of this shape should be? (You don't need an exact answer, but make a guess about what range you think they should be in or what, if any, relationship the volume and surface area should have with each other.)

   (c) Find the volume of Gabriel's Horn.

   (d) In general, a surface obtained by rotating an arc $y = f(x)$ on the interval $[a, b]$ about the x-axis has surface area $2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}dx$. Take a minute to consider on a sketch why this makes sense: What is the lateral surface area of a cylinder with radius r and length l? Consider an infinitesimal piece of the arc at a point x: what is its length and what is the radius of rotation at that point?

   (e) Find the surface area of Gabriel's Horn.

   (f) What do you notice about your answers? Discuss.

2. Homework: Do a little bit of research on Gabriel's Horn. When was it first considered by mathematicians? Which mathematician(s) are well known for working on this example? What are one or two famous explanations for resolving the apparant "paradox"? Can you find a way to think about this involving paint? One involving play doh?

   Write a paragraph explaining what you have found out (and as always, cite your sources), and bring it to turn in next time.