Calculus II ESP

Infinite Series

Worksheet # 16

Spring 2013

1. Identify which of the following are geometric series. For those that are identify the ratio of the series.

$$\sum_{n=0}^{\infty} \frac{7^n}{29^n} \qquad \sum_{n=0}^{\infty} \frac{1}{n^4} \qquad \sum_{n=0}^{\infty} \frac{n^2}{2^n} \qquad \sum_{n=5}^{\infty} \pi^{-n}$$

2. State the formula for the value of a geometric series and use it find the values of the following series or show that they diverge.

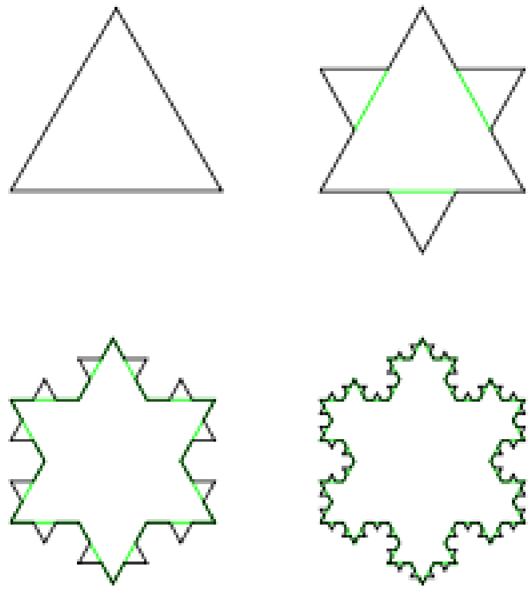
   (a) $1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots$

   (b) $\sum_{n=0}^{\infty} \frac{3}{11^n}$

   (c) $\sum_{n=0}^{\infty} \frac{3^n + 4^{n-2}}{5^n}$

   (d) $\sum_{n=0}^{\infty} \left(\frac{e}{2}\right)^n$

3. The fractal called the Koch Snowflake is constructed as follows: Start with I_0, an equilateral triangle of side length 1. The next figure in the sequence, I_{n+1} is obtained from figure I_n by replacing the middle third of each line segment in I_n with an "equilateral bump" (two sides of a triangle pointing outward) with sides of length 1/3 the length of the old segment. The limiting figure is the Koch Snowflake. The first four steps are illustrated below.



   (a) Let L_n be the perimeter of I_n. Show that $\lim_{n\to\infty} L_n = \infty$.
   (b) Let A_n be the area of I_n. Find $\lim_{n\to\infty} A_n$. It exists!
   (c) Homework: type "fractal" into a search engine and enjoy the pictures! (You might want to especially look for sites that let you zoom in on some more complicated fractals.) (You may also find yourself inspired to read more about fractals and the kind of math that can lead to getting and analyzing these types of pictures.)

4. Find the value of

$$\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$$

5. Imagine that the government of a small community decides to give a total of \$$W$, distributed equally, to all of its citizens. Suppose that each month each citizen saves a fraction p of her new wealth and spends the remaining fraction $1-p$ in the community. Assume no money leaves or enters the community, and all of the spent money is redistributed throughout the community.

(a) If this cycle of saving and spending continues for many months, how much money is eventually spent? Specifically, by what factor is the initial investment of \$$W$ increased? (Economists refer to this increase as the multiplier effect.)

(b) Evaluate the limits as $p \to 0$ and $p \to 1$ and interpret their meanings.