Calculus II ESP

Divergence and Integral Tests

Worksheet # 18

Spring 2013

1. We have learned two series by name: the geometric series and the p-series. Give a definition for each, and the conditions under which each one is convergent or divergent. For each of these types of series, give two examples, one which is convergent and one which is divergent.

2. (a) State the divergence test.

   (b) State the integral test.

   (c) Use the divergence test to check whether the series diverges. If this test is inconclusive, use the integral test.

   i. $\displaystyle\sum_{k=0}^{\infty} \frac{k}{2k+1}$

   ii. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

   iii. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+1}}$

   iv. $\displaystyle\sum_{k=1}^{\infty} ke^{-2k^2}$

3. Find a series matching the following description. Give some justification for your answer (analytic or numerical).

   (a) converges faster than $\sum \frac{1}{k^2}$ but slower than $\sum \frac{1}{k^3}$.

   (b) diverges faster than $\sum \frac{1}{k}$ but slower than $\frac{1}{\sqrt{k}}$.

   (c) converges faster than $\sum \frac{1}{k \ln^2 k}$ but slower than $\sum \frac{1}{k^2}$.

4. Given that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \pi^2/6$, and that the terms of this series may be rearranged without changing the value, determine the sum of the reciprocals of the squares of the odd positive integers.

   (The fact that the terms of this series may be rearranged without changing the value can be shown using the limit comparison test and properties of absolute convergence, which we will discuss soon. The fact that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \pi^2/6$ is known as the Basel problem and can be shown using Taylor series, which we will discuss in a couple weeks. For now, take these facts as given.)

5. Homework: For the next couple sections, you should have a list with you of special series and convergence tests. You'll need to memorize these in the end, but in the short run you should have a good reference with you all the time. Either make your own list of all the series and tests from sections 8.3-8.6, or make a copy of the list on page 584 of your textbook and have it with you whenever you are working on calculus (in particular, bring it to class next time). I recommend reading through the sections and making your own list since it will help you remember them, but a photocopy of the list in the book is better than no list!