Calculus II ESP

Series Convergence Tests

Worksheet # 19

Spring 2013

1. **Comparison Test**: State the Comparison Test, then use it to determine whether the infinite series converge or diverge.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 + \sqrt{n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 2^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

2. **Ratio Test**: State the Ratio Test, then use it to determine whether the infinite series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{5^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{4^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)!}$

3. **Root Test**: State the Root Test, then use it to determine whether the infinite series converge or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(5n - 3n^3)^n}{(7n^3 + 2)^n}$

4. **Limit Comparison Test**: State the Limit Comparison Test, then use it to determine whether the infinite series converge or diverge. (Note that the Limit Comparison Test is different than the Comparison Test.)

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n - n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 + 1}$

5. First, discuss which method to use in your group, and then decide whether the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{\sqrt{n^7+n-1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-3}}{4^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin \frac{1}{n}}{n^3}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^n}$

6. Prove that if $\sum a_k$ is a convergent series of positive terms, then $\sum a_k^2$ also converges.