Calculus II ESP **Spring 2013**

Taylor Polynomials

Worksheet # 22

1. Let f be a function with the first n derivatives defined at a. State the formula for the nth-order Taylor polynomial centered at a. How does this relate to the linear approximations we did in Calc I?

2. Find the second and third order Taylor polynomials centered at $x = a$ for the given function and value of a.

   (a) $f(x) = \ln(1 - x), \qquad a = 0$

   (b) $f(x) = \frac{1}{1+x}, \qquad a = 0$

   (c) $f(x) = \tan x, \qquad a = 0$

3. Consider $f(x) = \ln(1 - x)$ and its Taylor polynomials from the first problem.

   (a) Find a bound on the error in approximating $f(x)$ by $p_3(x)$ for values of x in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

   (b) Graph $y = |f(x) - p_3(x)|$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

   (c) At what points of $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is the error largest? Smallest?

   (d) Compare your results to the theoretical error bound you found in part a.