Calculus II ESP | **Spring 2013**

Taylor Series 2

Worksheet # 25

1. An essential function in statistics and the study of the normal distribution (bell curve) is the error function:
   $\text{erf}(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\, dt$
   (Recall that e^{-t^2} was our classic example of a function that cannot be anti-differentiated using elementary functions.)

   (a) Compute the derivative of erf(x)

   (b) Expand e^{-t^2} in a Maclaurin series, then integrate to find the first four nonzero terms of the Maclaurin series for erf.

   (c) Use the polynomial in part (b) to approximate erf(0.15) and erf(-0.09).

2. The expected (average) number of tosses of a fair coin required to obtain the first head is $\sum_1^\infty k\left(\frac{1}{2}\right)^{k-1}$. Find the value of this sum by finding a Taylor expansion for $\frac{1}{(1-x)^2}$. What is the most that you should be willing to pay to play a game where you flip a coin until you get a head, and are paid $1 for each flip?

3. The function $\text{Si}(x) = \displaystyle\int_0^x \frac{\sin t}{t}\, dt$ is called the sine integral function.

   (a) Expand the integrand in a Taylor series about 0.

   (b) Integrate the series to find a Taylor series for Si.

   (c) Approximate Si(0.5) and Si(1) using enough terms of your series to get an error less that 10^{-3}.

4. Using the binomial series formula, verify that

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \ldots, \quad \text{for } -1 < x \leq 1.$$

   Now use properties of power series, substitution and factoring to find the first four nonzero terms of the Taylor series centered at zero for $\sqrt{a^2+x^2}$ for $a > 0$. What is its interval of convergence? Write the series using sigma notation and binomial coefficients $\binom{p}{k}$