**Calculus II ESP** | **Spring 2013**

# Polar Coordinates

**Worksheet # 27**

1. Warm up:
   (a) Plot the points in polar coordinates: $(1, \pi/4), (3, 3\pi/4), (0, 0)$, $(-1, 0)$, $(1, -\pi/4)$
   (b) Given a point with cartesian coordinates $(x, y)$, give the equations that help find its polar coordinates $(r, \theta)$.
   (c) Given a point with polar coordinates $(r, \theta)$, give the equations to find its cartesian coordinates $(x, y)$.
   (d) Recall that polar coordinates are not unique. Give another set of polar coordinates for the following points.

   i. $(5, \frac{\pi}{4})$ ii. $(-2, \pi)$ iii. $(0, 0)$

2. Sketch graphs for the following equations in polar coordinates.

   (a) $r = 4\cos\theta$ (b) $r = 3 - 5\cos\theta$
   (c) $r = 3\cos 6\theta$ (d) $r^2 = 18\cos 2\theta$

3. Let $r = f(\theta)$ be a smooth function in polar coordinates.
   (a) Write the parametric form of this curve, with $\theta$ as your parameter.
   (b) Note that when we refer to the slope of this curve, we still mean the rate of change of the vertical with respect to the horizontal (*not* $dr/d\theta$). Show that the slope of $r = f(\theta)$ is

   $$\frac{dy}{dx} = \frac{f'(\theta)sin(\theta) + f(\theta)cos(\theta)}{f'(\theta)cos(\theta) - f(\theta)sin(\theta)}$$

   (c) Find the slopes of the lines tangent to the 4-petal rose given by $r = \sin 2\theta$ at the tips of its leaves. (Drawing a graph will tell you the answer, but use calculus to prove it.)

4. Find the area of the region inside the circle $r = 2cos(\theta)$ but outside the circle $r = 1$. Make a sketch and do your work in polar coordinates. Recall we did a similar problem in rectangular coordinates several weeks ago (the shape was called a lune). Was the problem simpler or more complicated in polar coordinates?