Math 165 Consumer's Surplus

This file is `http://www.math.uic.edu/~jlewis/math165/165demand.pdf`

Discussion of Consumer's Demand and Willingness to Spend

Consumer's Willingness to Spend is the Total amount $A(q)$ that consumers are willing to spend for q units. ($A(q)$ dollars)

The **consumer's demand function**, $p = D(q)$, is the rate of change of $A(q)$ wrt q; i.e., $D(q)$ is the **marginal willingness to spend**; units of $D(q)$ are dollars/unit.

$$D(q_0) \approx A(q_0 + 1) - A(q_0),$$

so that $D(q)$ approximates the price all consumers are willing to pay for the $(q_0 + 1)$st unit produced[1].

Note that

$$A(q_0) = \int_0^{q_0} D(q)\, dq.$$

In the geometric context, $\int_0^{q_0} D(q)\, dq$ represents the *area* under the graph of $p = D(q)$, and above the interval $0 \le q \le q_0$ in the q–axis.

The **Consumer's Surplus**, $CS(q_0)$ is the **total willingness to spend** – **actual expenditure for q_0 units at price** p_0.

$$CS = \int_0^{q_0} D(q)\, dq - p_0 q_0.$$

A **supply function**, $p = S(q)$ is the price at which all producers are willing to supply q units. It is generally assumed that $S(0) > 0$, and that $S(q)$ is an increasing function of q. The text examples, $S(q)$, are also concave upward, which reflects a typical assumption that the total cost function, $C(q)$, is concave upward.

I (JL) think of $S(q)$ as approximating the price required for the production of the $(q + 1)$st unit,

---

[1] If $1 \ll q_0$,

$$D(q) \approx \frac{A(q+1) - A(q-1)}{2},$$

so we might say that $D(q)$ approximates the price all consumers are willing to pay for the qth unit produced.

The **Producer's Surplus**, $PS(q_0)$, is the **total consumer expenditure for** q_0 **units at price** p_0 **– total amount producers receive for supplying** q_0 **units**.

$$PS = p_0 q_0 - \int_0^{q_0} D(q)\, dq.$$

The analysis is usually done for p_0 as the *equilibrium price* where supply equals demand. Solve the equation

$$D(q) = S(q).$$

The corresponding price, $p_0 = D(q_0) = S(q_0)$, is the *equilibrium price*. See Example 5.5.5 and Problems 5.5.15 – 5.5.19.

Another choice for p might be $p =$ the price for which profit is maximized. See Problems 5.5.33 and 5.5.34.