Math 165: Elasticity

If a quantity x is changed by an amount Δx, the *relative change in* x is the ratio $\dfrac{\Delta x}{x}$. The *percentage change in* x is $100\dfrac{\Delta x}{x}$. Note that there are *no units* for the ratio of two quantities with the same units.

Suppose the quantity q and the price p are related, e.g., by a relation of the form $q = D(p)$,

To understand the *price elasticity of demand*, take the ratio

$$\begin{aligned}\frac{\text{relative change in } q}{\text{relative change in } p} &= \left(\frac{\Delta q}{q}\right) \Big/ \left(\frac{\Delta p}{p}\right) \\ &= \frac{p}{q}\,\frac{\Delta q}{\Delta p} \\ &\to \frac{p}{q}\,\frac{dq}{dp}\end{aligned}$$

as $\Delta p \to 0$. Define

$$E(p) = \text{price elasticity of demand} \equiv \frac{p}{q}\,\frac{dq}{dp}.$$

Note that E(p) should be negative, and in general will depend on the value of the price p.

A practical interpretation of elasticity is that for every 1 percent *increase* in the price p, the demand q *decreases* by approximately $|E(p)|$ percent.

What is the significance of elasticity? The revenue $R = p \cdot q$, and

$$\begin{aligned}\frac{dR}{dp} &= 1 \cdot q + p \cdot \frac{dq}{dp} \\ &= q\left(1 + \frac{p}{q}\,\frac{dq}{dp}\right) \\ &= q\left(1 + E(p)\right).\end{aligned}$$

For $q > 0$, the sign of $\dfrac{dR}{dp}$ is the same as the sign of $1 + E(p)$.

There are three cases (Hoffmann, p. 246):

1. **Elastic Demand:** $|E(p)| > 1$, $1 + E(p) < 0$, $\dfrac{dR}{dp} < 0$, R is decreasing with respect to p. Demand is relatively sensitive to changes in price.

2. **Inelastic Demand:** $|E(p)| < 1$, $1 + E(p) > 0$, $\dfrac{dR}{dp} > 0$, R is increasing with respect to p. Demand is relatively insensitive to changes in price.

3. **Demand is of Unit Elasticity:** $|E(p)| = 1$, $1 + E(p) = 0$, $\dfrac{dR}{dp} = 0$, R has a critical number at p which is a likely relative maximum. The percentage changes in price and demand are approximately equal.

Exercises Section 3.4

23. $D(p) = -1.3p + 10$, $p = 4$.

$$\begin{aligned} \frac{dq}{dp} &= -1.3, \\ E(p) &= -1.3p/q, \\ E(p)\big|_{p=4} &= -1.08. \\ \frac{dR}{dp} &= -2.6p + 10 \\ \left.\frac{dR}{dp}\right|_{p=4} &= -.4 \end{aligned}$$

$|E(4)| = -0.59 < 1$, Inelastic Demand, R is decreasing with respect to p.

25. $D(p) = 200 - p^2$, $p = 10$.

$$\begin{aligned} \frac{dq}{dp} &= -2p, \\ E(p) &= -2p^2/q, \\ E(p)\big|_{p=10} &= -2. \\ \frac{dR}{dp} &= 200 - 3p^2 \\ \left.\frac{dR}{dp}\right|_{p=10} &= -100 \end{aligned}$$

$|E(4)| = 2 > 1$, Elastic Demand, R is decreasing with respect to p.

40. When an electronics store prices a certain brand of stereos at p hundred dollars per set, it is found that q sets will be sold each month, where $q^2 + 2p^2 = 41$.

a. Find the elasticity of demand for the stereos.

Using implicit differentiation, $2q\dfrac{dq}{dp} + 4p = 0$, so $\dfrac{dq}{dp} = \dfrac{-2p}{q}$, and $E(p) = \dfrac{-2p^2}{q^2}$.

b. For a unit price of $p = 4$ (\$400), is the demand elastic, inelastic, or of unit elasticity?

$E(4) = \dfrac{-2 \cdot 4^2}{3^2}$, Elastic Demand, R is decreasing with respect to p.