Math 165 Linear Price–Demand

Three Problems from `samplefinal2007.pdf`

2007 25. An apartment complex has 250 units. When the monthly rent for each unit is \$330, all units are occupied. Experience indicates that for each \$14 per month increase in rent, 4 units will become vacant. Each rented apartment costs the owner of the complex \$50 per month to maintain. What monthly rent should be charges to maximize profit? A) \$125.5 B) \$251 C) \$376.5 D)\$627.5

Solution Formula for demand is $q = 250(all) - (4 \text{ per } \$14) * (\text{increase in rent from}\$330)$.

$$\begin{aligned} q(p) &= 250 - (4/14) * (p - 330), \\ &= (2410/7) - (2/7)p \\ \text{Profit}(p) &= (p - 50) * q, \\ \frac{d\text{Profit}}{dp} &= (2510/7) - (4/7) * p \end{aligned}$$

The critical number is located at located at $p = 627.5$.

N.B. To express q as a linear function of p, we are actually using the point–slope form: Point $(p = 330, q = 250)$, slope $-4/14$.

26. A commuter's train carries 600 passengers each day from a suburb to a city. It now costs \$1 per person to ride the train. A study shows that 50 additional people will ride the train for each 5 cent reduction in fare. What fare should be charged in order to maximize total revenue? A) 78 cents B) 79 cents C) 80 cents D) 85 cents.

Solution: Formula for demand (riders) is $q = 600 - (50 \text{ per } .05) * (\text{increase in fare from}\$1.00)$.

$$\begin{aligned} q(p) &= 600 - (50/.05) * (p - 1.00), \\ &= 1600 - 1000p \\ R(p) &= p * q, \\ \frac{dR}{dp} &= 1600 - 2000p \end{aligned}$$

The critical number is located at located at $p = 0.80$.

28. A Florida citrus grower estimates that if 30 orange trees are planted, the average yield per tree will be 200 oranges. The average yield will decrease by 2 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield? A) 65 trees B) 15 trees C) 35 trees D) 60 trees

Solution: Let T = trees planted. Formula for *output per tree* (yield) is $Y = 200 - (2 \text{ per } 1 \text{ tree}) * (\text{increase in trees from } 30)$.

$$\begin{aligned} Y(T) &= 200 - (2/1) * (T - 30), \\ &= 260 - 2T. \\ R(T) &= T * Y, \\ &= 260T - 2T^2 \\ \frac{dR}{dT} &= 260 - 4T. \end{aligned}$$

The critical number is located at $T = 65$.