Math 165 Spring 2009: Special Assignment One

Due Wednesday, February 11, 2009, in Lecture

The Rules

- This assignment should be **typed**. For suggestions on typing, see
  `http://www2.math.uic.edu/~lewis/math165/165type.pdf`

- Special Assignment One is a GROUP PROJECT. All papers must be worked on and written up by groups of at least two and no more than four people. TYPE the group writeup. **Please use complete sentences to explain your work and answers.** For graphs, you may attach neat free hand sketches with enough labels for an outsider to understand the graph. Groups may assign tasks, but each member is responsible for understanding all parts of the assignment. The last paragraph should summarize the roles and activity of each group member.

- Turn in both the **WARMUP** and the **Main Course**.

- **Group Members may be in Different Sections or Lectures.**

- Every group should fill in and attach:
  `http://www2.math.uic.edu/~lewis/math165/165sa1groupmembers.pdf`.

WARMUP

W1. Differentiate with respect to x: $p(x) = \dfrac{1}{x^2+1}$.

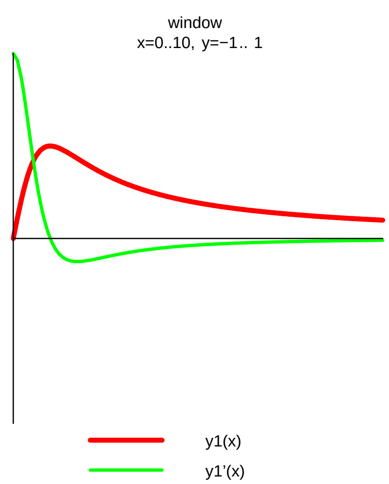
You may TYPE the expression as you might enter it on your calculator: `p(x) = 1/(x^2 + 1)`.

W2. Differentiate with respect to x: $r(x) = \dfrac{x}{x^2+1}$. Simplify your answer before proceeding.

W3. Find the critical numbers (Solve $r'(x) = 0$) and critical points of the graph of $r(x)$.

W4. Use your calculator to draw the graphs of $y1 = r(x)$ and $y2 = r'(x)$. A window of $0 \le x \le 10$, $-1 \le y \le 1$, is suggested.

Your graph should look like this:



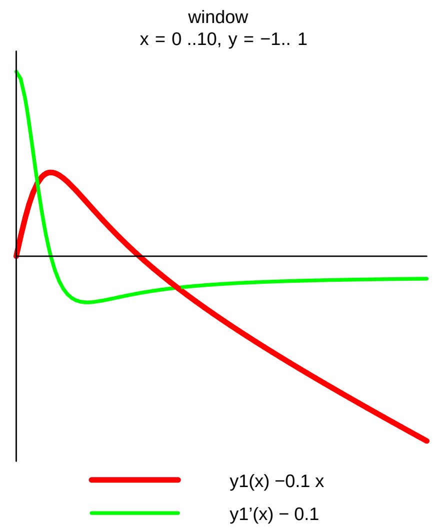
Notice that the graph of $y1 = r(x)$ has its MAXimum at the ROOT of $y2 = r'(x) = 0$.

W5. Now practice finding the positive root of the equation $y2 = 0$ by using your calculator. You should use several methods. Try TRACE AND ZOOM on the graph of $y2$. Use the ROOT or SOLVE or INTERSECT facility on your calculator. See your calculator manual. There are also several short manuals for TI calculators on the Blackboard Course Site under

`COURSE DOCUMENTS/Calculator Keystroke Guide.`

See also the *Calculator Introduction* in Hoffmann.

W6. Graph the functions $y1 = r(x) - 0.1x$ and $y2 = r'(x) - 0.1 = y1'$. A window of $0 \le x \le 10$, $-1 \le y \le 1$, is suggested. Your graph should look like this:



Notice that the graph of $y1 = r(x) - 0.1x$ has its MAXimum at the ROOT of $y2 = r'(x) - 0.1 = 0$.

W7. Using the methods of W5, now practice using your calculator to find the positive root of the equation $y2 = r'(x) - 0.1 = 0$. The solution should be about $x = .84$.

W8. Using the methods of W5, find the positive critical number for the function

$$\begin{aligned} y1 &= \frac{x}{x^2+1} - 0.1x \\ &= r(x) - 0.1x \end{aligned}$$

on the interval $0 \le x \le 10$. Use $y2 = y1'(x) = r'(x) - 0.1$. The critical number should be about 0.84.

Main Course: REVENUE COST PROFIT

This is based on Chapter 2 Section 2.3 Problem 48, p. 136, DEMAND AND REVENUE.

The manager of a company that produce graphing calculators determines that when x thousand calculators per month are produced, they will be sold when the price is

$$p(x) = 1,000/(0.3 * x^2 + 8)$$

dollars per calculator.

MC1. The revenue $R(x)$ in thousands of dollars derived from the sale of x thousand calculators per month is

$$R(x) = x * p(x).$$

We introduce a cost function. Assume that there is a fixed cost of 28000 dollars each month and that each calculator requires 31 dollars in parts, labor, etc. The cost function in thousands of dollars per month is

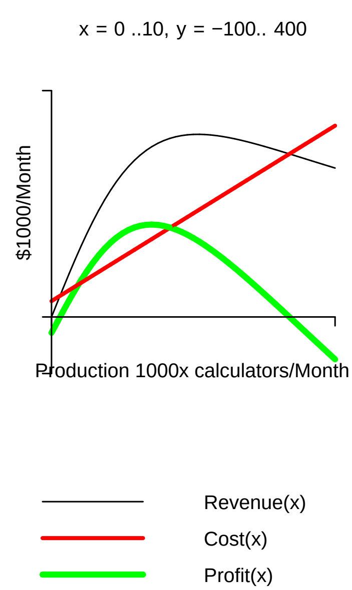
$$C(x) = 28.0 + 31.0 * x.$$

The profit $P(x)$ in thousands of dollars per month in terms of a production of x thousand calculators per month is

$$P(x) = R(x) - C(x).$$

Find a formula for $P(x)$.

MC2. Graph the functions $R(x)$, $C(x)$, and $P(x)$. A window of $0 \leq x \leq 10$, $-100 \leq y \leq 400$ is suggested. Your graph should look like this:



MC3. (BREAK EVEN POINT) What is the minimum number of calculators produced each month which assure a profit? Find the answer x to the nearest 0.01 (the nearest 10 calculators).

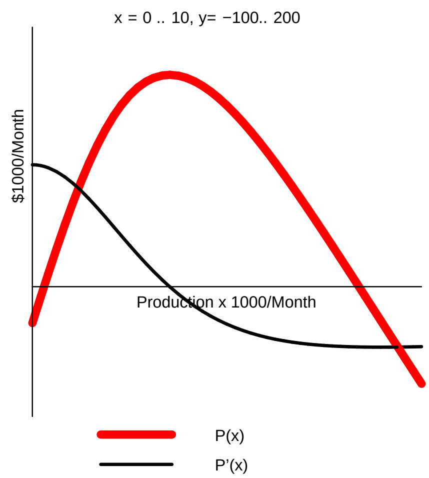
Hint: Use the calculator methods of W5 to solve the equation $P(x) = 0$ or $C(x) = R(x)$.

MC4. The way to maximize the profit, $P(x)$ is to find the critical number for the profit function. The derivative of the profit with respect to the demand x is the *marginal profit* function:

$$\frac{dP}{dx}.$$

Find a formula for the *marginal profit* function.

MC5. Graph the profit function, $P(x)$, and the marginal profit function, $P'(x)$. A window of $0 \le x \le 10$, $-100 \le y \le 200$. is suggested. Your graph should look like this:



MC6. (MAXIMUM PROFIT) The maximum profit results at the positive critical number of $P(x)$. Find the positive value of x for which

$$P'(x) = 0.$$

Estimate the monthly level of production x (nearest 0.01 thousand or 10 calculators) which yields the maximum profit.

Hint: Use the formula for the marginal profit function. Use the calculator methods of W5 to solve the equation

$$\frac{dP}{dx} = 0.$$

MC7. Summary:

(a) What level of production yields the maximum profit?

(b) What is the price (nearest dollar) of a calculator for the maximum profit?

(c) What is the maximum profit (nearest thousand dollars)?

Remember the Rules

The last paragraph of your typed writeup should summarize the roles and activity of each group member.