

# Functions

"Def" p. 2

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A function is a rule which assigns to each object in a set  $A$  (called domain of  $f$ ) exactly one object in a set  $B$  (called range)

N.B. - ~~range may be~~ Context - Range  $\left\{ \begin{array}{l} \text{exactly all} \\ \text{objects assigned} \\ \text{\& Maybe bigger} \end{array} \right.$

- Real nos.  
Convention: if not specified otherwise assumed to be all the nos for which defn/rule makes sense

e.g.  $y = \sqrt{x-1}$  domain  $\{x \mid x \geq 1\}$

$f(t) = \frac{1}{t}$  domain  $\{all t \neq 0\}$

(i.e) don't try sqrt(neg) divide by 0.

Compositions.  $G(g) = (80-g)^2$   
 $= f(g(g))$   $g(g) = 80-g$   
 $f(\cdot) = (\cdot)^2$

Example 1.1.5:

$p$  = price per item

$q, x$  = number of units  $q$  demanded  $x$  quantity

(produced, sold)

- consumers demand " $x$ " when  $p$  = unit price

$p = 51 - 0.27x$  ← function

← practical domain for  $ab$   
 $x > 0$  and  $51 - 0.27x > 0$

$x < \frac{51}{.27} \approx 188$

Cost of producing  $x$  tk

$C(x) = 2.23x^2 + 3.5x + 85$  thousand dollars

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$D(x)$

$(D(x))$

demand

$R(x) = \text{price/unit} \cdot \text{no. units}$   $(R(x))$

Revenue function

$P(x) = \text{profit}$   $R(x) - C(x)$   $(P(x))$

Same case on units  $x$  thousand [units]

$p$  dollars

~~$c$~~  thousand \$

(a)

$D(x) = -0.27x + 51$

$R(x) = (-0.27x + 51) \cdot x$  thousand doll's  
dollars/unit thousand

~~$C(x) = (-0.27x + 51) \cdot x - 0.27x + 51$  cost~~  
 ~~$P(x) = 2.23x^2 + 3.5x + 85 - (-0.27x + 51)$~~   
thousand \$

(b) Profitable when  $P(x) > 0$

$P(x) = -2.5x^2 + 47.5x - 85$

Get  $P(x) > 0 = \dots -2.5(x-2)(x-17)$

How? Quadratic Formula / Get Lucky Factor / Graph.

Profitable for  $2 < x < 17$ .

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### Example 1.10 Total Cost

Costs  $C(q)$

Total cost of manufacturing the first  $q$  units of "certain" items

$$\text{e.g. } C(q) = q^3 - 30q^2 + 500q + \underbrace{200}_{\text{startup cost}}$$

Realistic? Probably not.  
cost of producing

$$\text{"10th"} \quad C(10) - C(9)$$

$$[\text{Estimate } C(10 + \frac{1}{2}) - C(10 - \frac{1}{2})] [\text{Later}]$$

### Compositions. Practice 1.1. Prob 33

$$\frac{f(x+h) - f(x)}{h} \quad \left. \vphantom{\frac{f(x+h) - f(x)}{h}} \right\} \text{ difference quotient}$$

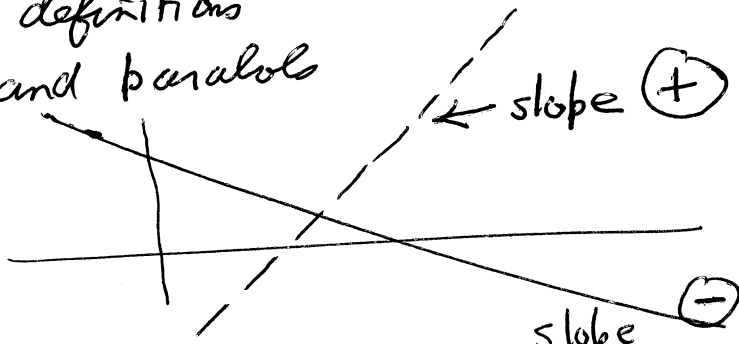
$$f(x) = \frac{1}{x} :$$

### Graphs: Visualize with calculator

Piecewise definitions

Lines and parabols

Line



-  $Ax + B$  (slope intercept form)

Point slope form:

Commuter train has 12000 riders/day  
fare is \$5. For each .10 increase in fare  
loses approx 300 riders.

Find a formula for

(1) Total number of riders at fare  $f$  ( $p$ ?)

(2) Revenue with fare at  $p$ .

slope/rate of change

$$= \frac{-300 \text{ riders}}{.10 \$}$$

$$Q = 12000 - \frac{300}{.10} (p - 5.00)$$

check units!