Limits: QUIZ THURSDAY $\leq$ Sec. 1.5

(p59) Defn $\lim_{x \to c} f(x) = L$ (finite number)

If ~~f~~ x gets close to c (but isn't = c), f(x) is close to (the finite number L)

N.B. i) close can be on either side of c

ii) actual value of f(c) does not matter; in fact, f(c) might not even be defined.

Examples

Polynomial, good formulas, etc

$$\lim_{x \to c} f(x) = f(c)$$

e.g. $\lim_{x \to c} \pi x^3 + 5x - A x^5 = \pi c^3 + 5c - Ac^5$

If there is any question at all about f(c) being defined, e.g. $\div 0$ or $\sqrt[n]{0}$ or $\sqrt{\text{negative}}$, investigation necessary

Rules (p. 61) Algebraic properties of limits

Sums "limit of $\overset{\text{diff}}{\text{sum}}$" = "sum of $\overset{\text{diff}}{\text{limits}}$"

more precisely: If $\lim_{x \to c} f(x) = L$ AND $\lim_{x \to c} g(x) = M$

[both finite nos.] then

$$\lim_{x \to c} (f \pm g)(x) = L \pm M.$$

Products "limit of products" = product of limits"

"Limit of Quotients" = "Quotients of Limits" PROVIDED that $\lim_{x \to c}$ (denominator) $\overset{\text{finite}}{\neq 0}$.

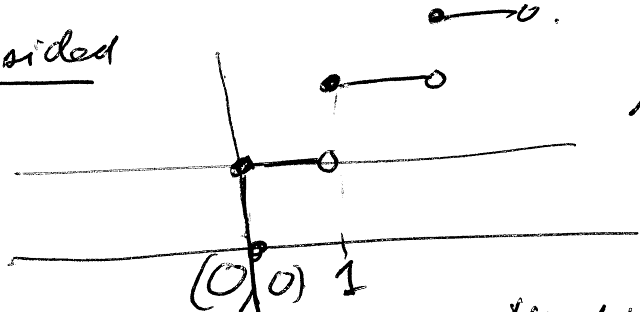
e.g. $\lim_{x\to \boxed{5}} \frac{(x-2)(x+4)}{(x-3)(x+5)}$

not 3 or -5.

$$= \frac{\lim(x-2)\cdot\lim(x+4)}{\lim(x-3)\cdot\lim(x+5)}$$

$$= \frac{(5-2)(5+4)}{(5-3)(5+5)}$$ (never dividing by 0 near $x=5$.

One sided



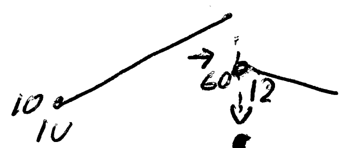
$\lim_{x\to1^-} f(x)=1$

$\lim_{x\to1^+} f(x)=2$

NO LIMIT.

If $\div 0$ investigate by rewriting to form which is not $\div 0$

Piecewise functions: Stock Price



$\lim_{t\to12^-(\text{mos})} f(x)=60$ $\quad \lim_{t\to12^+} =$ ⊔

$$\lim_{h\to0}\frac{(2+h)^2-2^2}{h} = \frac{(\cancel{(2+h)-2})(2+h+2)}{\cancel{h}} \to 4$$

$$\lim_{h\to0}\frac{(x+h)^2-x^2}{h}$$

note that "h" not "x" is changing and we get an answer which depends on x

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Sec. 1.6 One sided

(1) Stock Price

(2) $\sqrt{x}$ at $x=0$ (not reasonably defined for $x<0$)

$\div 0$ Ex 1.6.2 $\frac{x-2}{x-4}$ near $x=4$ At $x=4$ not defined $\div 0$

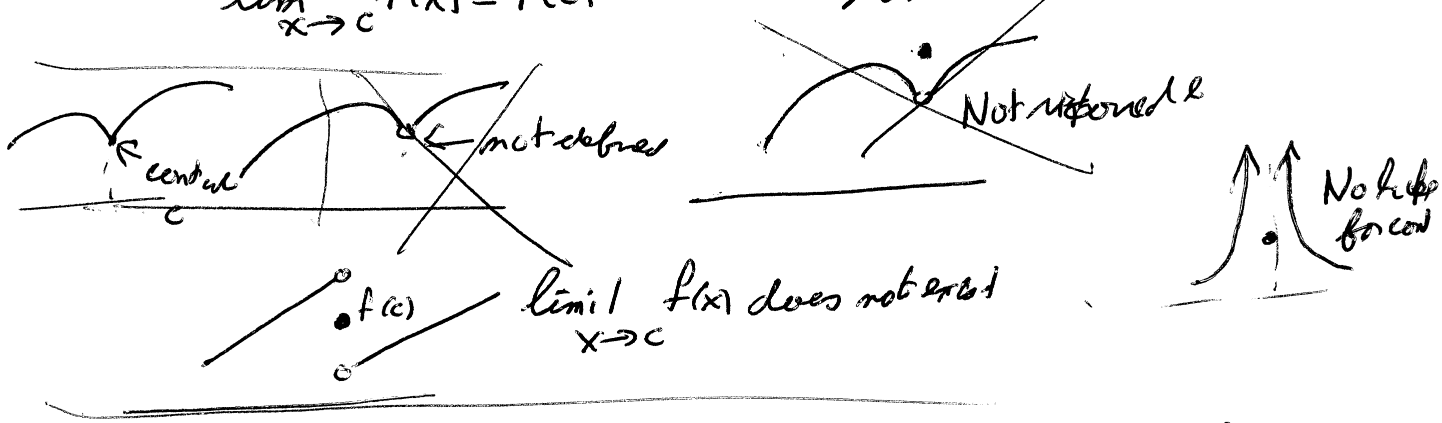
$x-2 \approx 4-2$ $[\lim_{x \to 2} x-2 = 2]$

$\frac{x-2}{x-4} \approx \frac{2}{x-4}$ Graph $\lim_{x \to 4^-} \frac{\cancel{(x-4)}\,2}{x-4} = -\infty$ [extended value]

lim (+∞ from right

A function f is continuous at c iff

- f(c) is defined c in domain
- $\lim_{x \to c} f(x) = f(c)$ } limit is what it should be



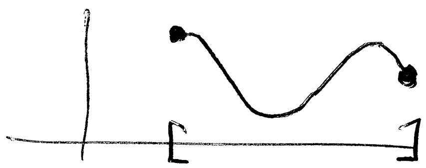
nice Algebraic formulas are continuous where defined

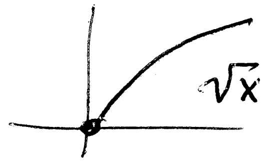
Thus $\frac{x^2 - 5x + 14}{(x-3)}$ is continuous at c for all $c \neq 3$
x for all $x \neq 3$

Continuity on an interval

"Continuous at all c in the interval
x
pts

Closed intervals are special p79





, $x \geq 0$

continuous inside
and appropriate one side lim at any included endpoint

Problem 1.6

(9) $\lim_{x \to 0^+} (x - \sqrt{x}) = 1 - 1$

forces $\lim_{x \to 0^+}$ only

(43) Postage

Piecewise $f(x) = \begin{cases} 2x^2 - x & x < 3 \\ 3 - x & x \geq 3 \end{cases}$

(N.B) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3} (2x^2 - x) \underset{\text{plug in}}{=} 2(3)^2 - 3 = 15$

$\lim_{x \to 3^+} (3 - x) = 0$ NOT cont

Cont at c, all $c \neq 3$