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$\frac{dx^p}{dx} = px^{p-1}$ $\frac{d}{dx}x = 1$ $\frac{dc}{dx} = 0$ (Power Rule)

$\frac{du^p}{du} = pu^{p-1}$ ...

Derivative // Rate of Change // s = position at time t $\frac{ds}{dt}$ = velocity (with sign)

Relative Rate of change of $Q(x)$

Approx $\frac{\Delta Q}{Q} / \Delta x \to \frac{\frac{dQ}{dx}}{Q(x)}$ (Percentage ---- mult by 100

relative change

Example 2.2.7 (billion\$ [per year]) (t years after 1995)

GDP $= t^2 + 5t + 106$

Rate of change in 2005 ($t = 10$)

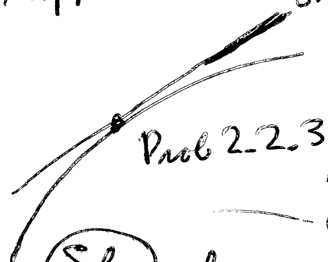
$\frac{dGDP}{dt} = 2t + 5$, when $t=10$ $\frac{dGDP}{dt} = 2\cdot 10 + 5 = 25$ billion

Relative Rate $\frac{25 \text{ billion}}{256 \text{ billion}} \approx .10 = 10\%$ (units per year if GDP is thought of as "per year" - annualized)

$10^2 + 5\cdot 10 + 106$

Tangent line to graph of $f(x)$ at point $(c, f(c))$

[linear approx] slope $f'(c)$ or $\frac{dy}{dx}$ EVAL at $x=c$ WRITTEN $\left.\frac{dy}{dx}\right|_{x=c}$



Prob 2.2.35 Tangent line to

$y = -2x^3 + \frac{1}{x^2}$; $x = -1$

Soln $\frac{dy}{dx} = -6x^2 - \frac{2}{x^3}$ [Why power formula $\frac{d}{dx}x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

when $x = -1$ $\left.\frac{dy}{dx}\right|_{x=-1} = -6 + 2 = -4$

Point $y = \cancel{8+1=9}\ 2 + 1 = 3$

$y = 3 - 4(x - (-1)) = \cancel{3}\,{-1} - 4x$

GRAPH: (~~Got~~ Graph muddled by vert asymptote at $x=0$

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2.2 Prob. 57: Annual Earnings ($1000)

$A(t) = 0.1t^2 + 10t + 20$ t years after founding in 2000

— Rate $\frac{dA}{dt}$ when 2004 ($t=4$

$\frac{dA}{dt} = 0.2t + 10 \approx 10.8$ thousand$/yr

$$\%\text{Rate} = \frac{\frac{dA}{dt}}{A} \cdot 100 = 100 \cdot \frac{10.8}{61.6} \approx 18\%$$

$A(4)\ 0.1 \cdot 4^2 + 10 \cdot 4 + 20$

$\approx 1.6 + 40 + 20 = 61.6$

2.3 Product Rules

Algebraic Rules	Fn	Der.	Common
$\pm$	$Au \pm Bv$	$Au' \pm Bv'$	$A\frac{du}{dx} \pm B\frac{dv}{dx}$
PRODUCT	uv	$u'v + uv'$	$\left(v \cdot \frac{1}{v}\right)' = 1' = 0 = \frac{v'}{v} + v\left(\frac{1}{v}\right)'$
QUOTIENT	$\frac{1}{v}$	$-\frac{v'}{v^2}$	$\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$
	$\left(\frac{u}{v}\right)'$	$\frac{vu' - uv'}{v^2}$	

Example 2.3.1. $P(x) = (x-1)(3x-2)$ $[= 3x^2 - 5x + 2$

$\frac{dP}{dx}$ [Product Rule] $= 1(3x-2) + (x-1)3$ $[= 6x - 5$

$\frac{dP}{dx} = \frac{d}{dx}[3x^2 - 5x + 2] = 6x - 5$

Ex 2.3.3 Things which are product Revenue $x \cdot p$ $x =$ quantity ~~sold~~ demanded at price x

Suppose Demand is $x(t) = t^2 + 3t$ (hundred pieces) t months (quantity/month)

price is $p(t) = -2t^{3/2} + 30$

$R(t) = x(t) + p(t) = (t^2 + 3t)(-2t^{3/2} + 30)$

$$\frac{dR}{dt} = (2t+3)(-2t^{3/2} + 30) + (t^2 + 3t)(-3t^{1/2})$$

when $t = 4$ $\frac{dR}{dt} = 11(-16 + 30) + 28(-6) = -14$

$R'(4)$ is negative

Example of product

2.3. problem 5 $f(x) = \frac{1}{3}(x^5 - 2x^3 + 1)\left(x - \frac{1}{x}\right)$

$$f'(x) = \frac{1}{3}\left\{(5x^4 - 6x^2)\left(x - \frac{1}{x}\right) + (x^5 - 2x^3 + 1)\left(1 - \frac{1}{x^2}\right)\right\}$$

const

neg powers

$$\frac{d}{dx}\frac{1}{x^p} = -\frac{p}{x^{p+1}}$$

Quotient $\left(\frac{u}{v}\right)' = \frac{\qquad}{\qquad}$

$\frac{\qquad}{v^2}$ (one over " v^2 ")

$\frac{v \qquad}{v^2}$ ← v on top

$\frac{vu' \qquad}{v^2}$ ← vu'

$\frac{vu' - uv'}{v^2}$ ← fill in the rest

Prob 7 $y = \frac{x+1}{x-2}$

$$\frac{dy}{dx} = \frac{(x-2)(1) - (x+1)\cdot 1}{(x-2)^2} \underset{\text{sim}}{=} \frac{x-2-x-1}{(x-2)^2} = \frac{-3}{(x-2)^2}$$