20090223 1/

Optimization

3.4: Revenue, Profit Average Cost

Opt function on an interval [a,b] of y (nice)

occurs only at { CRITICAL NOS / Special including endpoints a,b

3.4.2 $S(t) = t^3 - 10.5t^2 + 30t + 20 \quad 1 \le t \le 6$

Max & Min

$\frac{dS}{dt} = ~~2t - 21t~~ + 30$

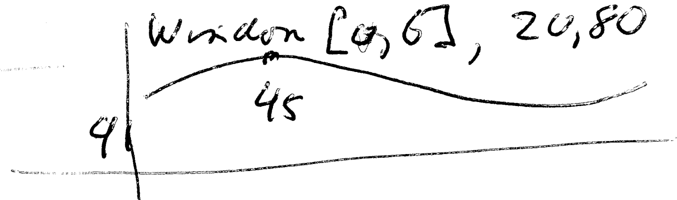
$= ~~2t(t-15)~~(t-2)$

$= 3t^2 - 21t + 30 = 3(t^2 - 7 + 10) = (t-2)(t-5)$

CR NOS $t=2$, $t=5$ both in [1,6]

$S'' = 6t - 21$ { $t=2$ NEG ∩ REL MAX; $t=6$ >0 ∪ REL MAX

Window [0,6], 20,80



Still Check ends ≃41, ≃45, 32, 37

1, 2, 5, 6

40.5, 46, 32.5, 38

Example 3.4.5: $C(q) =$ formula $0.4q^2 + 3q + 40$ (\$1000)

$p = p(q) = 22.2 - 1.2q$

Max Profit $P = q \cdot p - C(q)$

$= q(22.2 - 1.2)q - [0.4q^2 + 3q + 40]$

$= q(22.2 - 1.2) - [0.4q^2 + 3q + 40]$

$= q^2[~~22.2~~ - 1.2 - .4] + q[22.2 - 3] - 40$

$= ~~-1.6q^2 + 22q - 40~~ \; -1.6q^2 + 19.2q - 40$

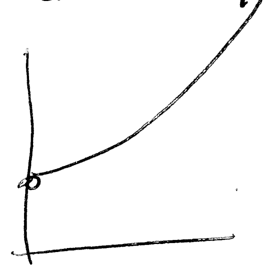
Quadratic { Roots $\frac{dP}{dq} = -3.2q + 19.2$

$= 0$ when $q = \frac{19.2}{3.2} =$

$q = 6$

When is Average Cost = Marginal Cost 20090223 2

Average Cost = $\frac{C(q)}{q}$



(N.B.) As $q \downarrow 0$ average cost $\uparrow +\infty$

$C(q) = -0.4q^2 + 3q + 40$

$A(q) \frac{d}{dq}\left(\frac{C(q)}{q}\right) = +0.4q + 3 + \frac{40}{q}$

$\frac{dAC}{dq} = +0.4 - \frac{40}{q^2}$, $q^2 = \frac{40}{.4} = 100$,

$q = 10$ (-10 thrown out)

Marginal Analysis Cut for maximum profit

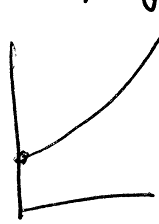
$\frac{dP}{dq} = 0$ OR $R'(q) = C'(q)$

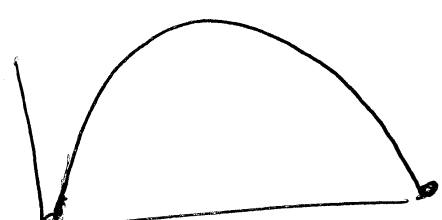
Marginal Analysis for Minimal Average Cost

$A(q) = \frac{C(q)}{q}$ is minimized when

$A(q) = C'(q)$

Picture





Average Revenue

$\frac{R(q)}{q}$ → slope at

Average Profit

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3.4.9. $g(x) = x + \frac{1}{x}$ on $\frac{1}{2} \le x \le 3$

Variation $[2, 3]$

Elasticity of Demand.

Price Elasticity of Demand [Elasticity of Demand with respect to p.

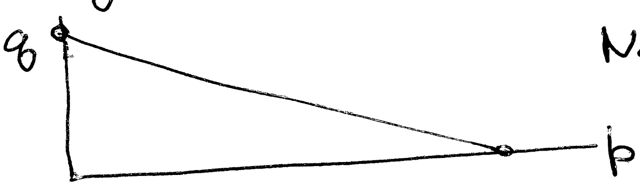
p elasticity of q

$$\simeq \frac{\%\text{ change in } q}{\%\text{ change of } p} \approx \frac{100\,\frac{\Delta q}{q}}{100\,\frac{\Delta p}{p}} \approx \boxed{\frac{p}{q}\frac{dq}{dp}}$$

$\equiv$ price elasticity of demand (usually negative

[Example 3.4.6] $q = 240 - 2p$ N.B. $0 \le p \le 20$

Not usual picture



Revenue $q \cdot p$. $\frac{dR}{dp} = \frac{dq}{dp} \cdot p + q = 0$

when $\frac{dq}{dp}\frac{p}{q} + 1 = 0$

$E(p) = -1$ Unit Elasticity

$E(p) < -1$ [$|E(p)|$] Elastic Demand

$E(p) > -1$ [$|E(p)| < 1$] inelastic