20090302 4

<u>Elasticity</u>: q and p related (N.B. assume $p>0, q>0$)

$$E(p) \equiv \lim_{\Delta p \to 0} \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \quad \begin{matrix} \} \text{ [\%] rel change in } q \\ \text{due to} \\ \} \text{ [\%] relative change in } p \end{matrix}$$

$$= \frac{p}{q}\frac{dq}{dp} \quad \text{(NEGATIVE)}$$

"price elasticity of demand" "~~of~~ demand elasticity wrt. p"

Measures ratio of "small" % changes" $E(p)$

~~Expand~~ <u>Expect</u> $|E(p)| < 1$

$R = p \cdot q$ is INCREASIN for $d p$

$\Delta R \simeq (p+\Delta p)\cancel{-p}(q+\Delta q)$

"demand is inelastic [wrt p]"

$|E(p)| = 1$ demand of unit elasticity ← possible max R

$|E(p)| > 1$ "demand is elastic" wrt p

LINEAR PRICE DEMAND

3.4.23 $q = -.3p + 10$; $|E(p)|$ at $p = 4$

$\boxed{\frac{dq}{dp} = -.3}$ when $p = 4$, $q = 10 - 1.2 = 8.8$

$E(p) = \frac{p}{q}\frac{dq}{dp} = \frac{4}{8.8}(-.3) = -\frac{1.2}{8.8} \approx -.136 <$ $|\quad| < 1$

$R = p \cdot q = p(-0.3p + 10)$

... Maximum at $\frac{10}{.3} \div 2 \approx$ = $\frac{10}{.6} \approx$

(N.B.) $|E(p)| = 1$ when $\frac{p}{10-.3p}(.3) = 1 \approx \underline{16.6}$

$.3p = 10 - .3p$

$.6p = 10$, $p \approx \underline{\underline{16.6}}$.

Relation to max Rev

$$q = D(p)$$

$$R = p\cdot q, \quad \frac{dR}{dp} = q + p\frac{dq}{dp} = q\,(1+E(p))$$

Sign of $\frac{dR}{dp}$ is same as that of $1+E(p)$ recall $q>0$

15, 19, 23, 27, 29

$\boxed{\text{Prob. 23}}$ Maximum Possible volume of cylindrical can made from 27π in^2 of material.

~~Fixed constraint~~ Variables r, h

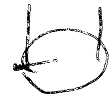
— Constraint Area = constant

$A(r,h) = 2\pi r^2 + 2\pi r h = 27\pi$

top & bottom

Prob 23 Constraint $\cancel{2\pi r h + \pi r}$

Prob. 23 $A = \underbrace{2\pi r^2}_{\text{bot + top}} + 2\pi r h = 27\pi$



N.B. $2\pi r h = 27\pi - 2\pi r^2$

$$h = \frac{27 - 2r^2}{2r} = \frac{27}{2r} - r$$

$$V = \pi r^2 h = \pi\left(\frac{27}{2}r - r^3\right)$$

$$\frac{dV}{dr} = \pi\left(\frac{27}{2} - 3r^2\right); \quad r^2 = \frac{9}{2} \quad r = \frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{2}$$

$$h = \frac{27}{2\cdot 3/\sqrt{2}} = \frac{27}{2r} - r = \dots = \frac{27\sqrt{2}}{2\cdot 3} - r = \frac{9}{2}\sqrt{2}$$

$$\cancel{= 3r??} = 3\sqrt{2} = \boxed{2r}$$

Actual Volume $\pi r^2 h = \pi\left(\frac{9}{4}\cdot 2\right)\cdot 3\sqrt{2}$

$$= \frac{27}{2}\sqrt{2}.$$

Inventory prob 29 [See 5A2 and Example 3.5.7

Order 800/yr order cost \$10/order

Storage cost (.40/bot)/yr

Order x times

$$\text{Cost} = \text{cost of } 800 + \$10\cdot \underbrace{x}_{\text{no. of orders}} + 800\cdot \underbrace{\frac{1}{2x}}_{\text{average storage time}}\cdot .40$$

$$= 800 * 20 + 10x + \frac{800\cdot .40}{2\ x}$$

$$\frac{dC}{dx} = \$10 - \frac{800\cdot .40}{2x^2}$$

$$x^2 = \frac{800\cdot .40}{2\cdot 10} = 40 * .40 = 16, \quad x = 4; \text{ bottles } \frac{800}{4}$$