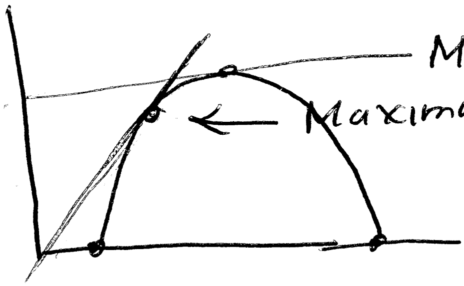
Test One Friday – Sishtla in L.C. Cl.   20090309   ✓

$\leq$ 4.2 (previous stuff assumed)

Problem of the day:

Avg Profit $\frac{P(q)}{q}$

(use rule)



Exp and ln

Also Avg cost Problem: $\frac{C(q)}{q}$

Solve $b^? = x$ $a>0, a\neq 1$ (usually $a>1$)

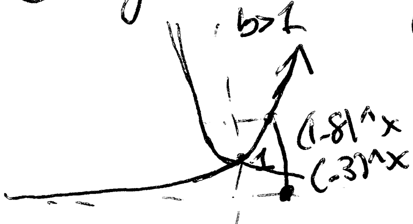
notation text

Solution for x POSITIVE

"range" of a^x is POSITIVE NOS

Can you solve $b^x = 2$ CALCULATOR

(the answer was positive)



$b<1$ (e.g. $\frac{1}{2}$)

The function b^x has range (image) all POSITIVE real numbers

b fixed POS, $\neq 1$

Logarithm base b

$\log_b(x)$ is the number which answers the question

$b^? = x$.

(The INVERSE fn to the "base b exponent fn" b^x.

$\boxed{\text{N.B.}}$ (tautology) $b^{(\log_b x)} = x$ (for $x>0$)

(1.8)^x

~~Calculator~~

20090309 2/

Special Base e [$\approx$ ...] and base 10 [computation

$e^{?} = x$ called log-base-e $\log_e(x) = \ln(x)$

[Literature Base: ln, $\log_e$, "log" sometime

[Base 10]: $\log_{10}$, "log" most of the time

Calculator $\boxed{LN}$ and it's INVERSE FN $\exp(x) = e^\wedge x = e^{x} = e^\wedge(x)$

~~LOG~~ SPECIAL PROPERTIES (stated for ln — "true for all base")

$\log_b b = 1$ ($b^{?} = b$) $\ln(e) = \ln e = 1$

$\log_b 1 = 0$ $b^{?} = 1$ answer 0

$\ln(\text{product}) = \text{sum of } \ln$

$\ln(r \cdot s) = \ln(r) + \ln(s)$

PROD $\longrightarrow$ +

$\ln(r/s) = \ln(r) - \ln(s)$

QUOT $\longleftrightarrow$ $-$

constant POWERS $\to$ "constant multiple"

$\ln(x^r) = r\ln(x)$

$\ln(x^r)$ answers the question $\cancel{x}e^{?} = x$

well. $e^{[r\ln(x)]} = e^{[\ln(x)]r} = [\underbrace{e^{[\ln(x)]}}_{\text{defn. log}}]^r = x^r$

Graph is reflection to $y = x$

(can calculator use window "Z quad"

$\boxed{\text{VARIABLES} > 0}$

logs make sense only for no. > 0

log(negative no.) delicate — depends on context!

Inverse relationship

$$e^{\ln(x)} = \exp(\ln(x)) = x \quad \boxed{x>0}$$

$$\ln(e^x) = x \quad \boxed{\text{all } x.}$$

Solving equations: En "Pure exponentials" on one on one side (No sums!)

$$3 = e^{20x}$$

Solution $\boxed{\text{both sides} > 0 \text{ we hope}}$

ln of both sides

$$\ln(3) = \ln(e^{20x})$$
$$= 20x(\ln(e))$$

simplification rules for ln

$$= 20(x)$$
$$x = \frac{\ln(3)}{20}$$

Verify: numerically / or definition

Conversion formulas $\ln_b a = \frac{\ln(a)}{\ln(b)}$ Don't remember

unless you are sure you will not make a mis*

All other $\log_b$ can be expressed "in terms of $\ln(x)$"

Solve $a^x = b$ (assumed a, b, pos $a \neq 1$)

"ln" $\ln(a^x) = \ln(b)$

$$x(\ln(a)) = \ln(b) \quad x = \frac{\ln(b)}{\ln(a)}$$

(actually this is above

CC doubling Time

$$FV(t) = FV(0)e^{rt}$$

Doubles when $\frac{FV(t)}{FV(0)} = 2 = e^{rt}$ ... $t = \frac{\ln(2)}{r}$

$$\ln(2) \approx \frac{.693}{r}; \quad \frac{.70}{.08}$$ (Rule of 7's

Exponential Growth / Decay

doubling time / half-life

"Simplify" $\ln\left(\frac{1}{x}+\frac{1}{x^2}\right)$

LCD $= \ln\left(\frac{x+1}{x^2}\right) = \ln(x+1) - \ln(x^2) = \ln(x+1) - 2\ln(x)$

$\boxed{49}$ Variation

$C(t) = 0.4(\cancel{2 - 0.13}e^{-.02t} \leftarrow 0.4(2 - .13e^{-.02t})$ important

Solve $C(t) = .75$

$.75 = 0.4(2 - .13e^{-.02t})$

ln immediately does not work

$.75 = (0.4)(2 - .13e^{-.02t})$

$(.75 - .4\cdot 2) = -.4(.13)e^{-.02t}$

Now take ln both sides