Antiderivatives: fn → deriv of fn (well posed - formulas ...)

Given a derivative ~~fn~~ (rate of change) function, what function(s) ~~is the given~~ has / have given as derivative

— Antiderivatives on an interval; i.e. don't try to go across special points ($\div 0$, etc.)

Usually write $f(x)$ [t] and denote an antiderivative as
↑ lower case

$F(x)$ [t]
↑ upper case

An antiderivative of $2x+1$ is x^2+x (alt. interval "all x")

An antid of $\frac{1}{x}$ is $\ln(x)$, $x>0$

An antid of $\frac{1}{x}$ is $\ln(|x|)=\ln(-x)$, $x<0$

Exercise antid of $\frac{1}{x}$ is

But also antid of $2x+1$ is $x^2+x+\begin{matrix}1\\ [5]\\ [\pi]\end{matrix}$

antid of $\frac{1}{x}$ is $\ln(x)+83.5$ $(x>0)$

Most general antiD of $f(x) = \underbrace{F(x)}_{\text{particular antiD}} + \underbrace{C}_{\text{any constant } C}$

Reconciliation

$f(x) = 2(x-1)$ antiD $(x-1)^2$ works

$f(x) = 2x-2$ antiD x^2-2x works

N.B. $(x-1)^2 = x^2-2x+1 = x^2-2x \underbrace{+1}_{\text{"+C"}}$

Integral [Indefinite Integral] "Notation"

p. 364 9e If $F'(x) = f(x)$ [on an interval]

"the integral of "f(x)" "dx = F(x)+C

$$\int f(x)\,dx = F(x)+C$$

↑ integral sign ↑ f(x) or a formula of x ↑ the "variable of integration

$$\int f(t)\,dt = F(t)+C$$

$\int f(x)\,dt \overset{?}{=}$

Some formulas $\int k\,dx = kx + C$

power rule $\int x^a\,dx = \frac{1}{a+1}x^{a+1} + C$, $a \neq -1$

pp.365 Careful $a<0$ "C" may differ for $x<0$ and $x>0$

$\int \frac{1}{x}\,dx = \ln(x) + C$ $x>0$ > don't use together

$= \ln(-x) + C$ $x<0$ $\longrightarrow$ $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$ $a \neq 0$

~~You~~ Algebraic ~~Therearesum~~ ~~A~~ $Af(x) \pm Bg(x)$ rules

[Products are Different matter / Quotient and rule

(N.B.) $\int x^2 \cancel{dx}\ 5x^3\,dx = \int x^2\,dx + \int 5x^3\,dx$

$= \frac{x^3}{3} + 5\cdot\frac{x^4}{4} + C$

Only one "C" needed

Initial Value Problem

$\underbrace{\frac{dy}{dx} = 3x^2+1}_{\text{Diff Eqn (DE)}}$; $\underbrace{y = 6 \text{ when } x = 2}_{\text{Initial Cond. } y|_{x=2} = 6}$ "y at $x=2$ is 6.

y is "one of" $\int 3x^2+1\,dx = \cancel{6x+}x^3 + x + C$

$y(2)=6$ $2^3+2+C=6$ $C=-4$

$y = x^3 + x - 4.$

Exam 5.1.5. $C(q)$

Marginal Cost $\frac{dC}{dq} = 3q^2 - 60q + 40$

Cost of producing first two units 900

Cost of first 5 units

first q units ($q > 2$?)

$$C(q) = \int 3q^2 - 60q + 400\ dq$$

$$= \ldots q^3 - \frac{60q^3}{2} + 400q \boxed{+C}$$

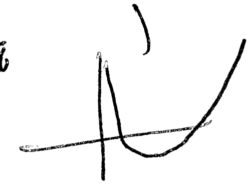
$$C(2) = 900 = 2^3 - 30 \cdot 2^2 + 800 + C$$

$$\ldots C = 212$$

(N.B. Critical numbers of q at

$3q^2 - 60q + 40$ factor? $(3q\quad)(q$

$$q = \frac{60 \pm \sqrt{(60)^2 - 480}}{6}$$ two positive roots



not realistic!

N.B Fixed formulas ~~are~~ for antiderivatives are tricky and may be "impossible"

AntiD practice

$\boxed{\#5}$ $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C,$ $x > 0$, $x < 0$] and both

#11 $\int (3t^2 - \sqrt{5t} + 2)\ dt = t^3 + \sqrt{5}\frac{t^{3/2}}{3/2} + 2t + C$, $t > 0$.

↑ fault

#23 $\int \sqrt{t}\,(t^2 - 1)\ dt =$ $\int t^{2.5} - t^{.5}\ dt = \frac{t^{3.5}}{3.5} - \frac{t^{1.5}}{1.5} + C$

NO PRODUCT RULE

$t > 0$

$\boxed{31}$ $\frac{dy}{dx} = 3x - 2$ $y = 2$ when $x = 1$

$$y = \int 3x - 2\ dx = 3\frac{x^2}{2} - 2x + C$$

$$y(1) = 2 = 3\frac{1^2}{2} - 2 \cdot 1 + C, \ldots C = \frac{5}{2}$$

$$y(x) = \frac{3}{2}x^2 - 2x + \frac{5}{2}$$